

# Economics Lecture 10

2016-17

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# Course Outline

## 1 Consumer theory and its applications

1.1 Preferences and utility

1.2 Utility maximization and uncompensated demand

1.3 Expenditure minimization and compensated demand

1.4 Price changes and welfare

1.5 Labour supply, taxes and benefits

1.6 Saving and borrowing

## 2 Firms, costs and profit maximization

2.1 Firms and costs

2.2 Profit maximization and costs for a price taking firm

## 3. Industrial organization

3.1 Perfect competition and monopoly

3.2 Oligopoly and games

# 3.1 Perfect competition and monopoly

# 3.1 Perfect competition and monopoly

1. Introduction to industrial organization
2. Markets with a fixed number of price taking firms
3. Firms with different costs & economic rent
4. Welfare economics of a tax with supply and demand
5. Welfare economics of a subsidy with supply and demand
6. Monopoly
7. Cartels

# 1. Introduction to industrial organization

One homogeneous good of known quality, no product differentiation

One price for all buyers and sellers

No asymmetric information, buyers know the characteristics of the good and the price charged by each firm.

No transactions costs, e.g.. search, negotiation, legal fees, bid ask spread ....

## Market structures

1. perfect competition – price taking firms
2. monopoly – one firm that affects price
3. oligopoly – a small number of firms that affect price

## 2. Markets with a fixed number of price taking firms

Perfect competition implies price taking,

nothing a firm does affects the prices it gets for output and  
pays for inputs,

nothing a purchaser does affects the price it pays for  
output.

[See previous lecture](#)



# When is price taking plausible?

A homogeneous good, all firms produce an identical product.

Large number of buyers and sellers each with a small market share.

Everyone can observe prices.

Note:

Price taking does not imply that prices do not change.

Price taking does imply that an individual buyer or seller cannot do anything to change prices.

[See previous lecture](#)

$p = MC$  follows from



Profit maximization implies firms equalize

Profit maximization does **not** imply that firms produce the level of

*See previous lecture*

$p = MC$  follows from **profit maximization**.

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Profit maximization implies firms equalize **marginal cost of production to price given a surplus per unit of production**.

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*See previous lecture*

$p = MC$  follows from **profit maximization**.

Profit maximization implies firms equalize **marginal cost of production to price given a surplus per unit of production**.

Profit maximization does **not** imply that firms produce the level of **output that minimizes AC**.

*See previous lecture*

Markets with a fixed number  
of price taking firms

# Markets with price taking firms

Assume for now a fixed set of firms price taking profit maximizing firms.

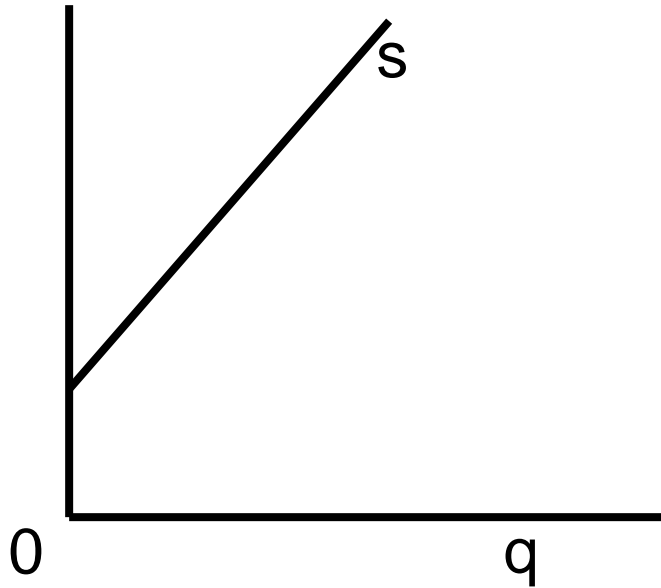
In an equilibrium with a fixed set of price taking firms each firm supplies a profit maximizing level of output given input and output prices

The market clears, that is **supply = demand**.

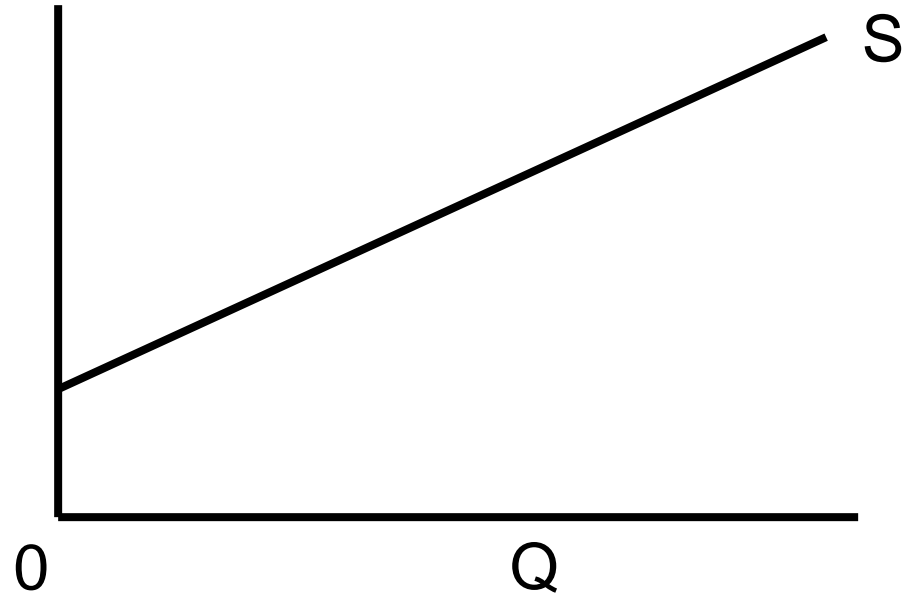
The next slides show how to solve the model.

Supply and demand are shown as straight lines for simplicity only.

## How to solve this model



firm

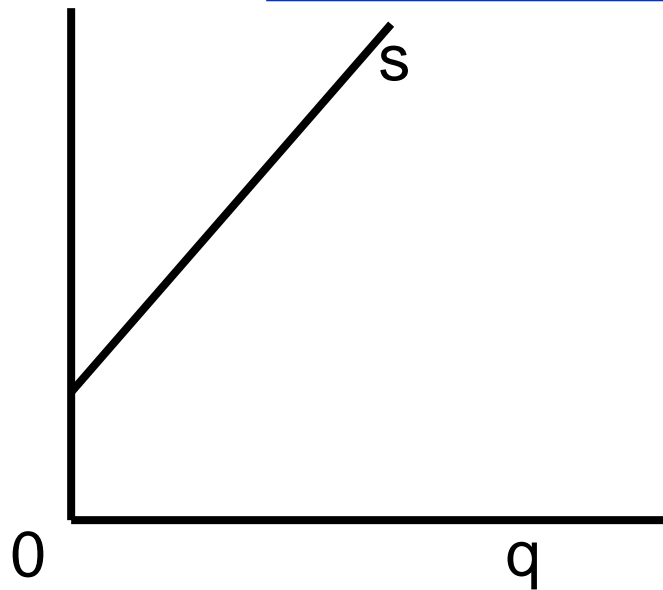


industry

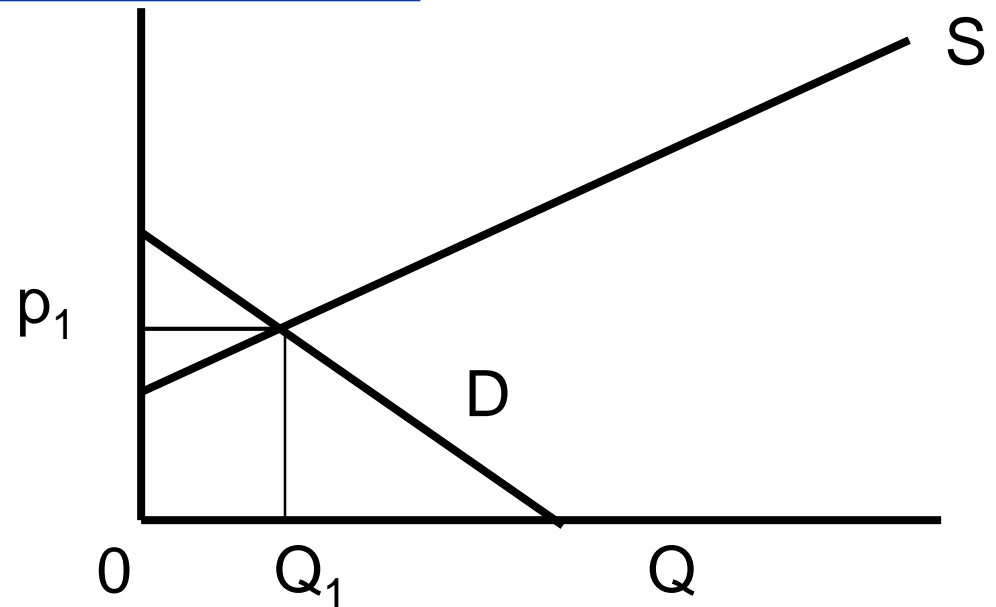
industry supply = horizontal sum of firm supply



## How to solve this model



firm

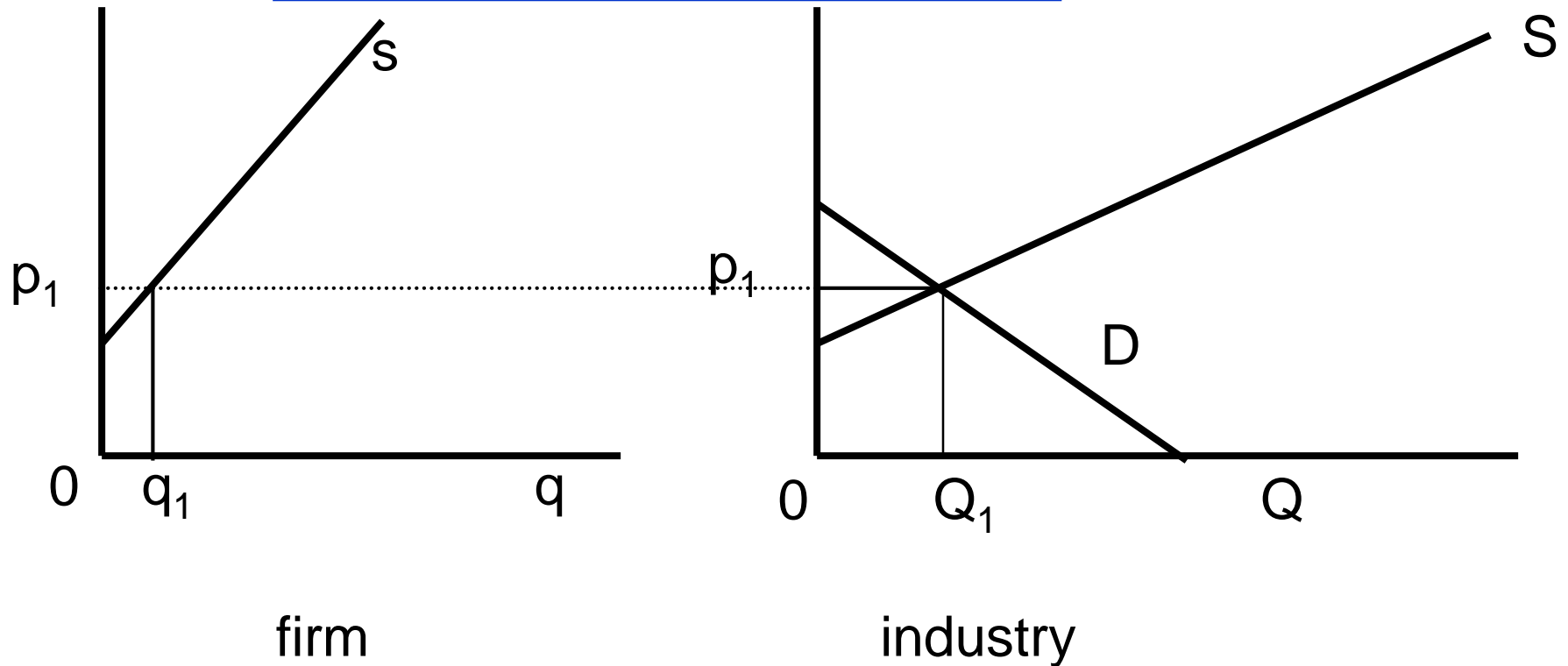


industry

industry supply = horizontal sum of firm supply

Find industry price  $p_1$  and quantity  $Q_1$  by intersection of supply and demand on industry diagram.

## How to solve this model

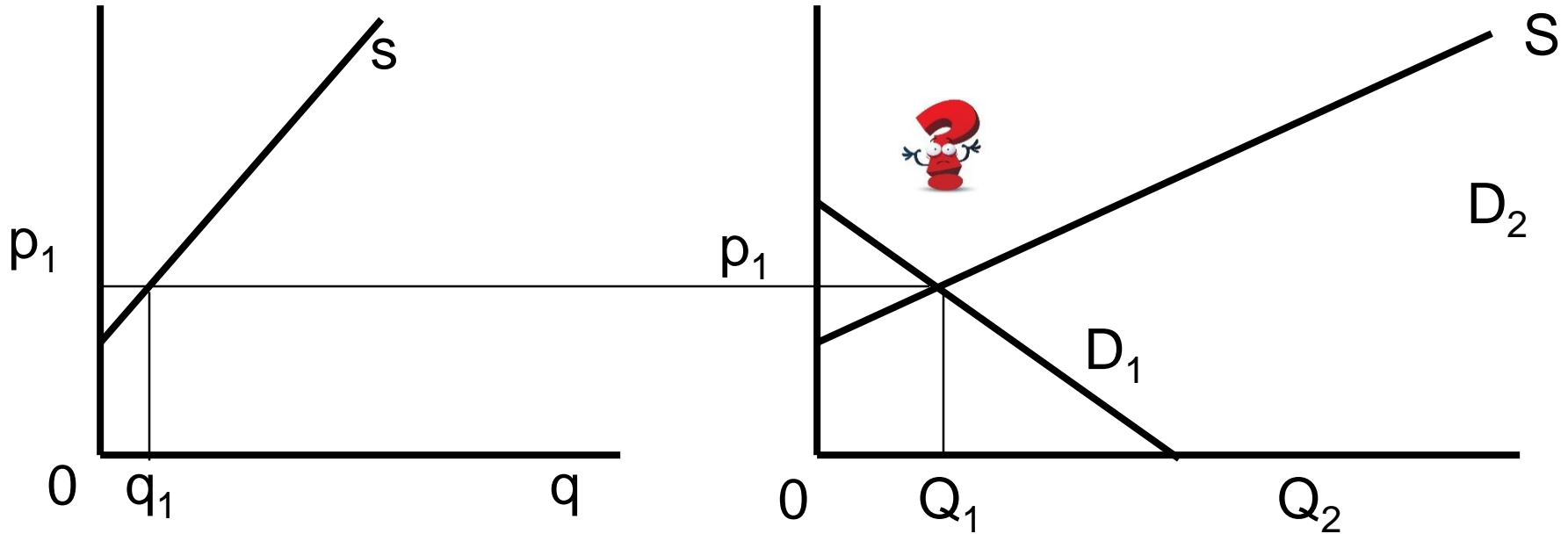


industry supply = horizontal sum of firm supply

Find industry price  $p_1$  and quantity  $Q_1$  by intersection of supply and demand on industry diagram.

Given the industry price find firm output  $q_1$  on firm diagram.

# Increase in demand

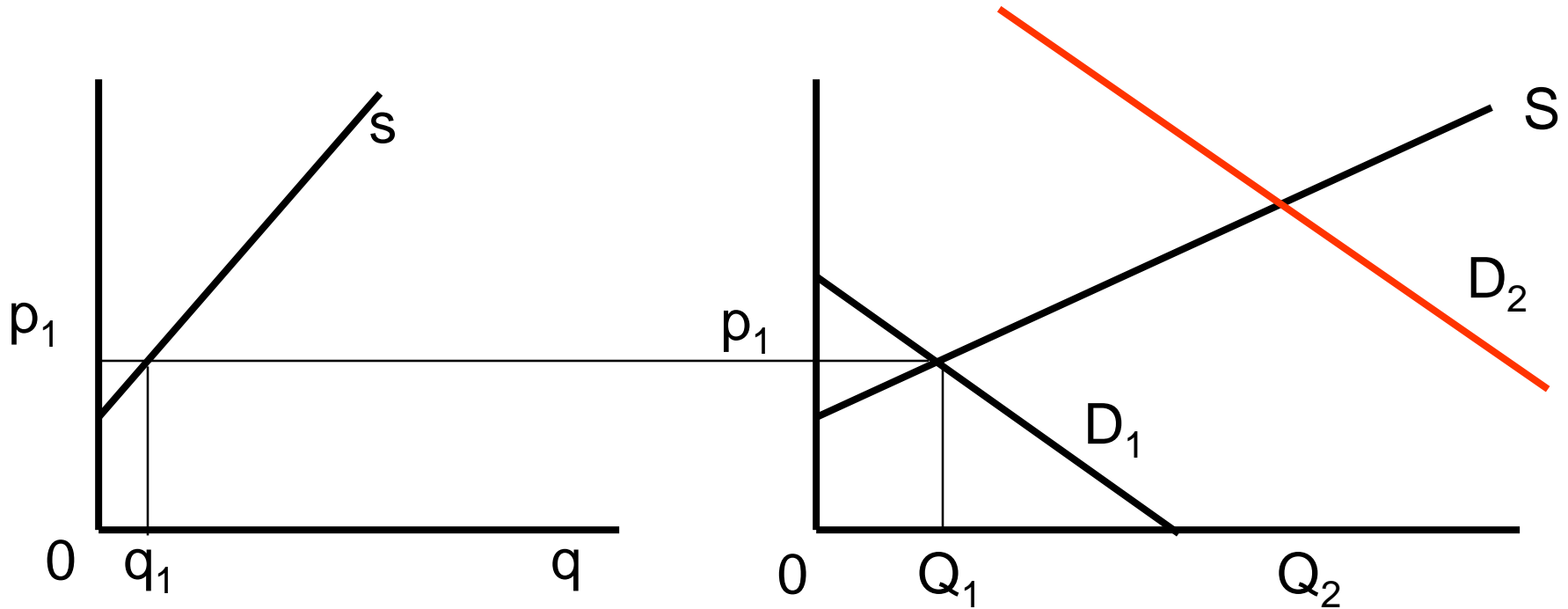




Demand curve shifts ( demand at each price)

Industry diagram price from  $p_1$  to ,  $Q$  from  $Q_1$  to .

Firm diagram firm quantity from  $q_1$  to .

# Increase in demand

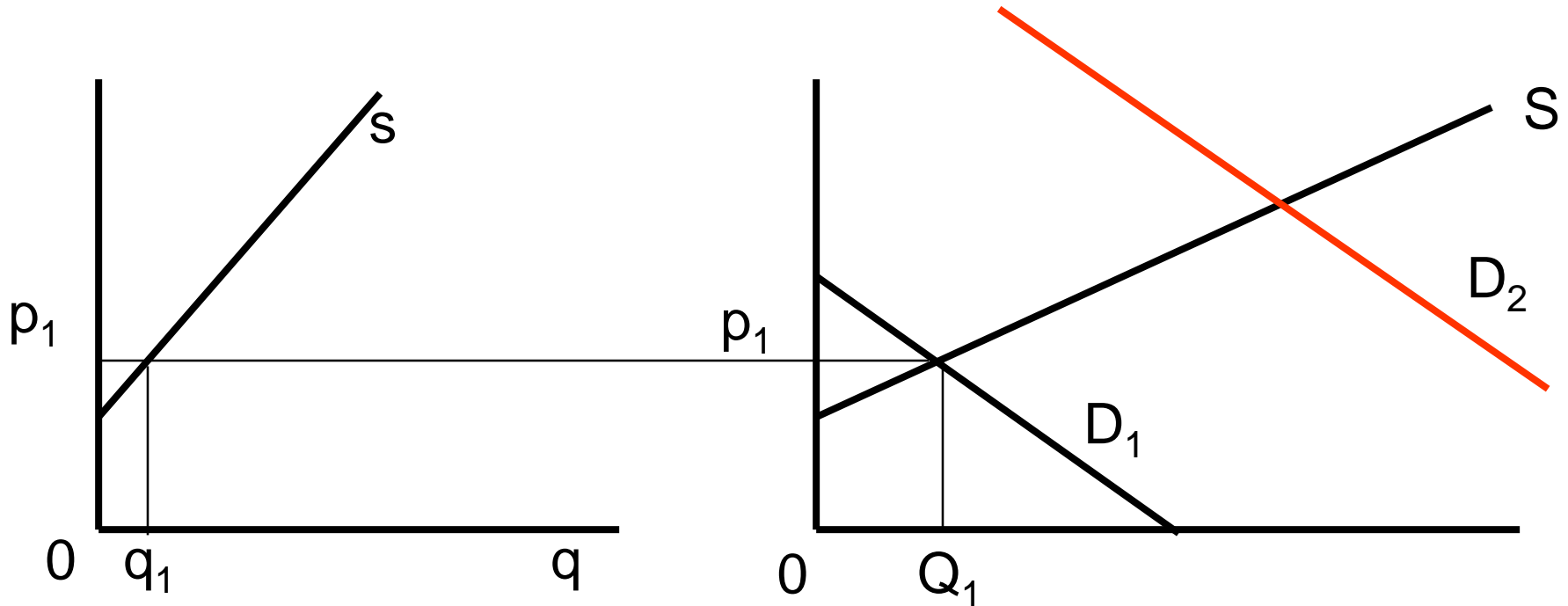


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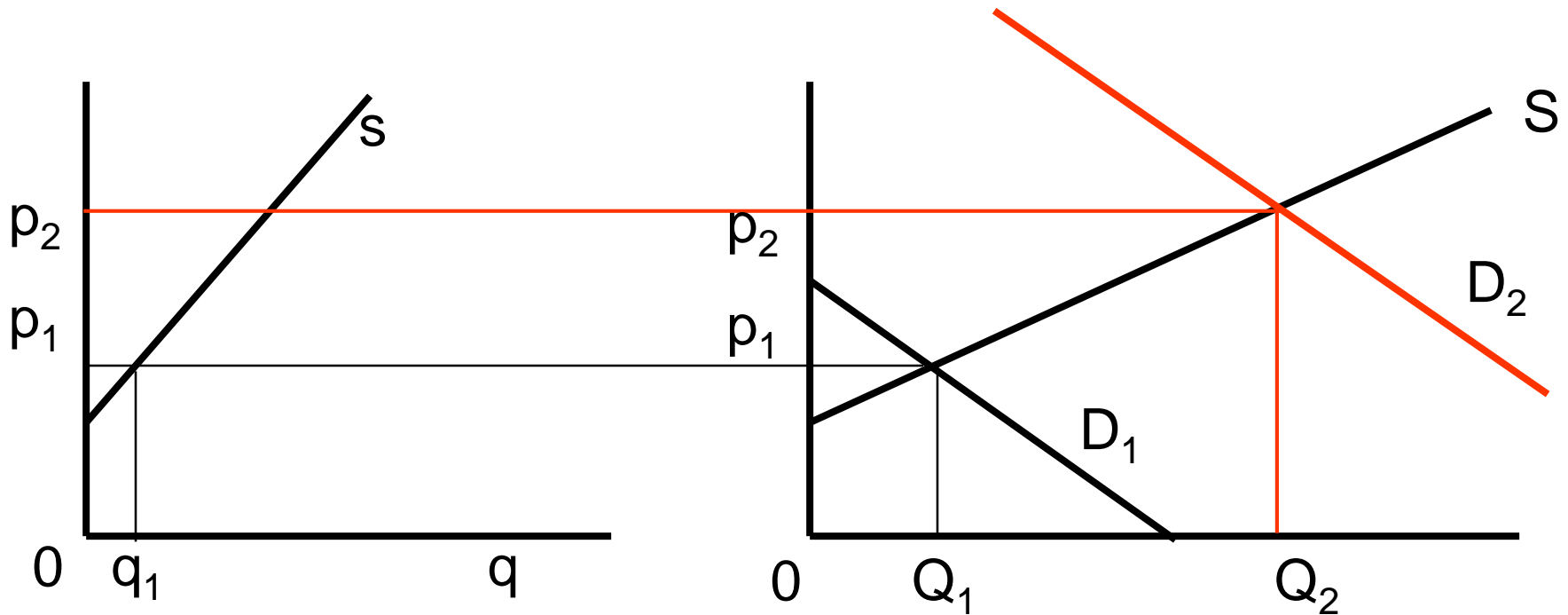


Demand curve shifts **out** (**higher** demand at each price)

Industry diagram price 🤖 from  $p_1$  to  $p_2$ ,  $Q$  🤖 from  $Q_1$  to  $Q_2$ .

Firm diagram firm quantity from  $q_1$  to .

# Increase in demand

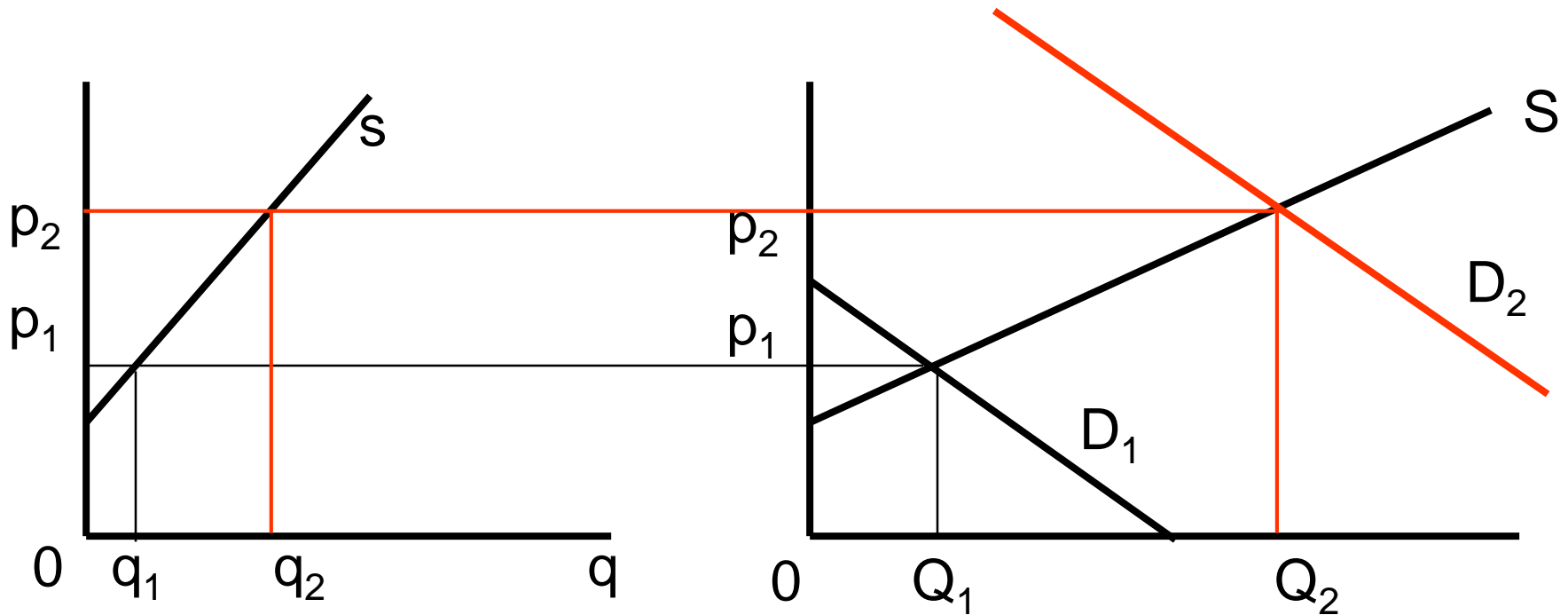


Demand curve shifts **out** (**higher** demand at each price)

Industry diagram price **↑** from  $p_1$  to  $p_2$ ,  $Q$  **↑** from  $Q_1$  to  $Q_2$ .

Firm diagram firm quantity **?** from  $q_1$  to  $q_2$ .

# Increase in demand

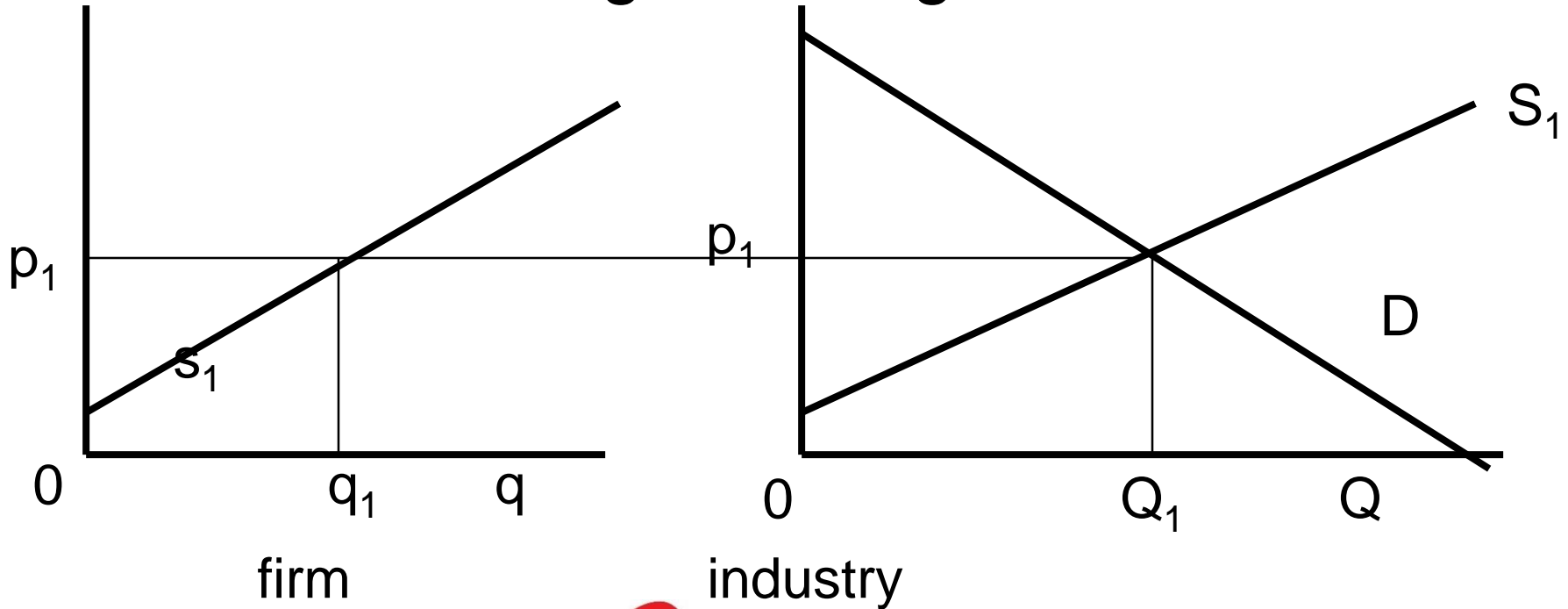


Demand curve shifts **out** (**higher** demand at each price)

Industry diagram price  $\uparrow$  from  $p_1$  to  $p_2$ ,  $Q \uparrow$  from  $Q_1$  to  $Q_2$ .

Firm diagram firm quantity  $\uparrow$  from  $q_1$  to  $q_2$ .

# Change in marginal cost



Increase in input prices  MC.

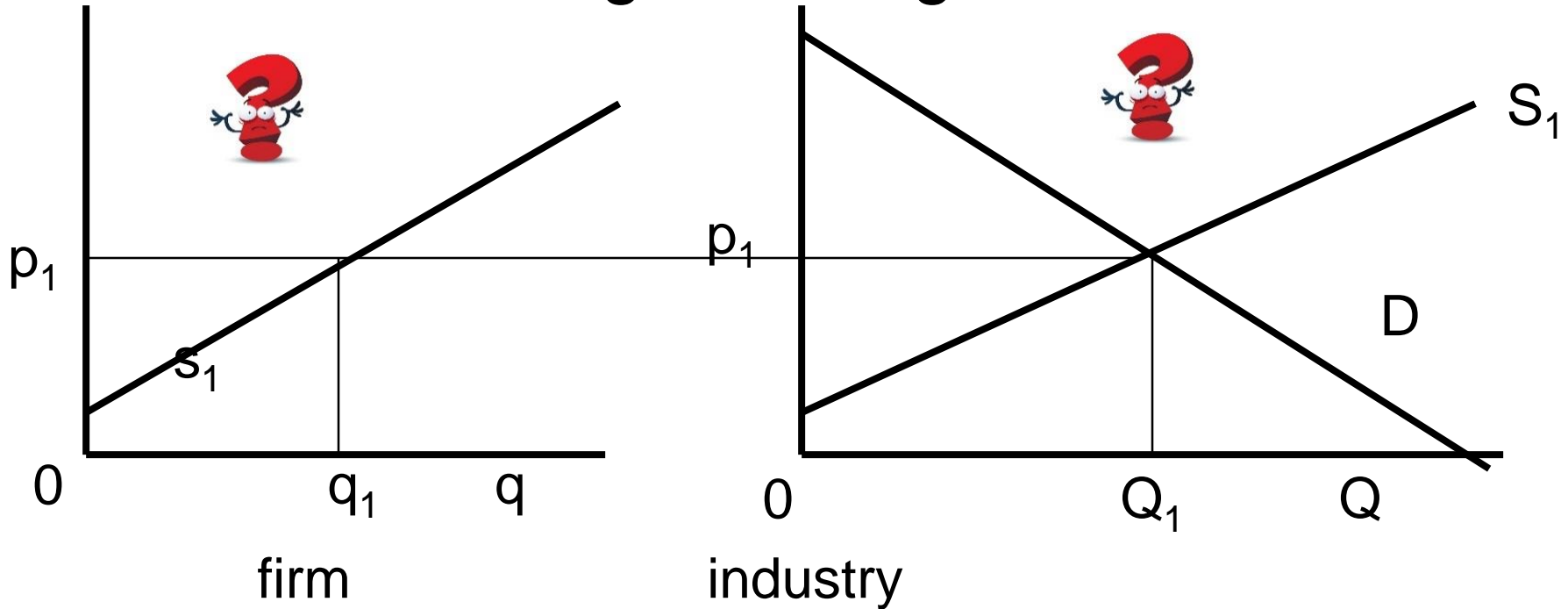
Supply curves  $s_1$  &  $S_1$  move

Industry diagram price from  $p_1$  to ,  $Q$  from  $Q_1$  to .


Firm diagram firm quantity from  $q_1$  to .



# Change in marginal cost



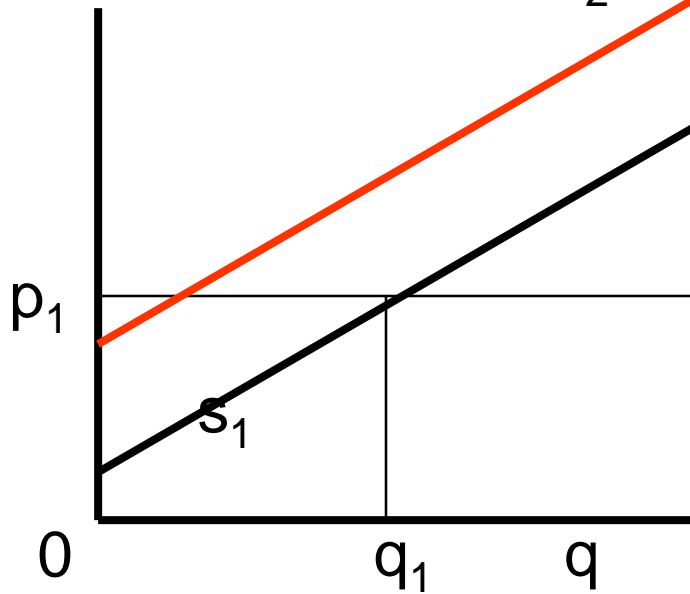
Increase in input prices  $\uparrow$  MC.

Supply curves  $s_1$  &  $S_1$  move  to  $s_2$  &  $S_2$ .

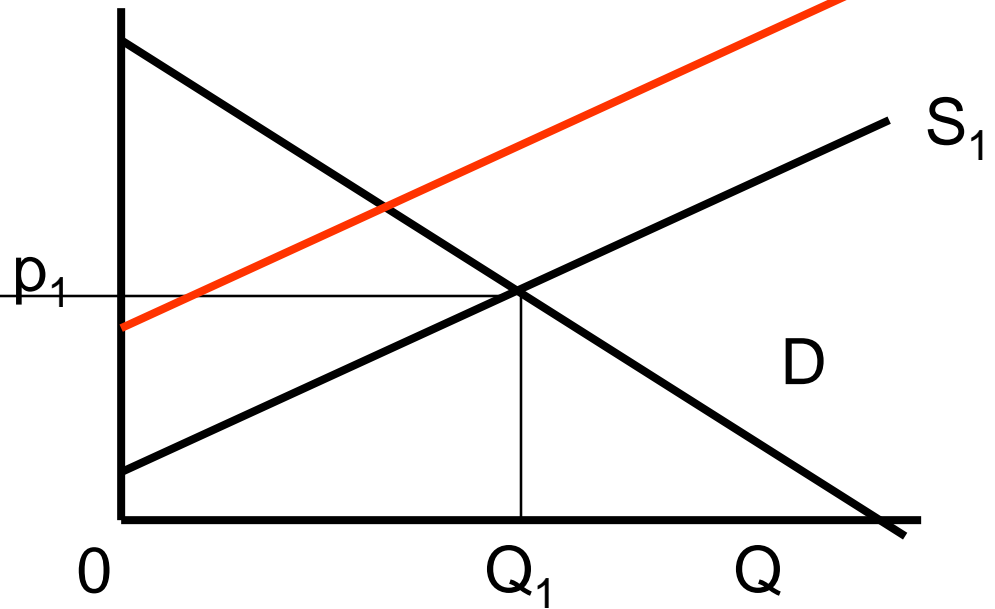
Industry diagram price from  $p_1$  to ,  $Q$  from  $Q_1$  to .

Firm diagram firm quantity from  $q_1$  to .

# Change in marginal cost



firm



industry

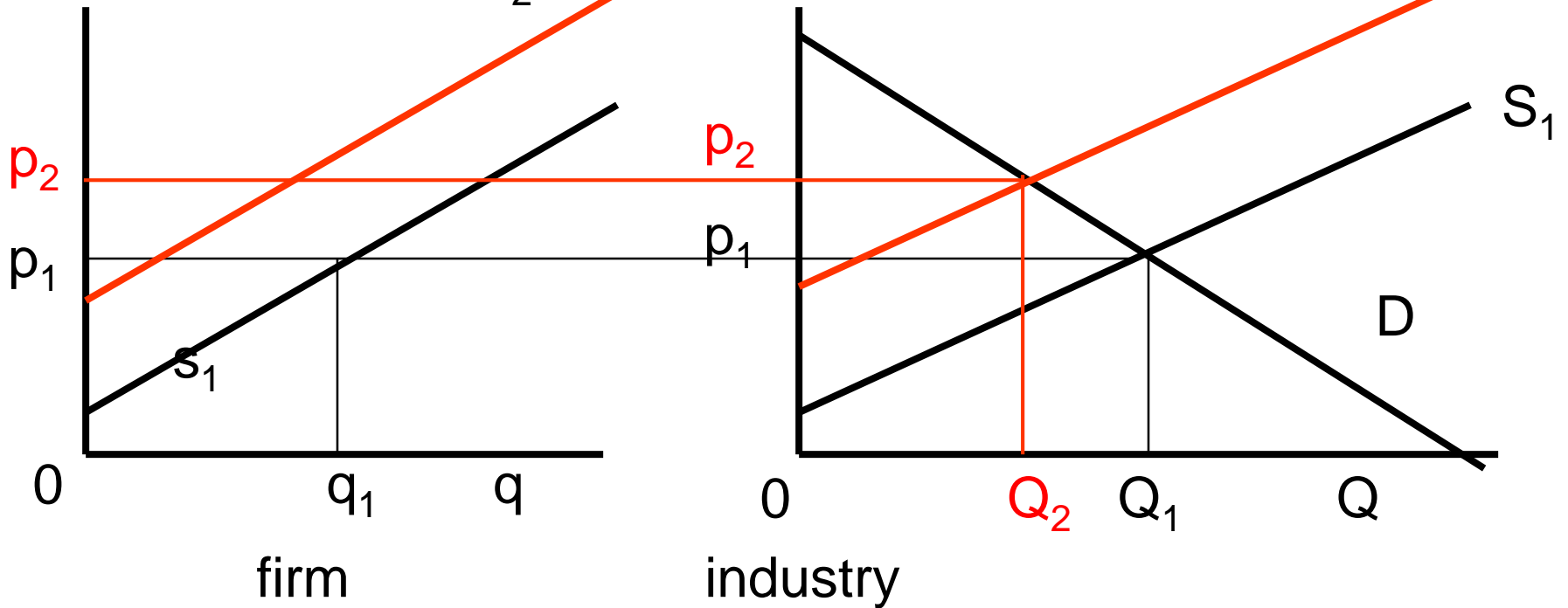
Increase in input prices  $\uparrow$  MC.

Supply curves  $s_1$  &  $S_1$  move up to  $s_2$  &  $S_2$ .

Industry diagram price  $\uparrow$  from  $p_1$  to  $p_2$ ,  $Q$   $\downarrow$  from  $Q_1$  to  $Q_2$ .

Firm diagram firm quantity from  $q_1$  to  $q_2$ .

# Change in marginal cost



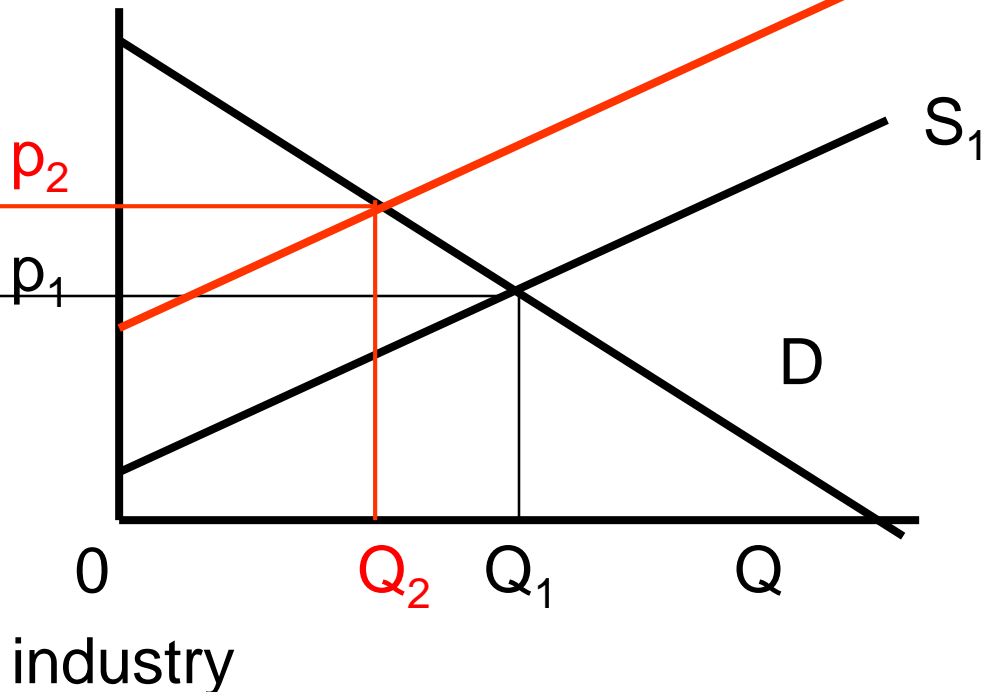
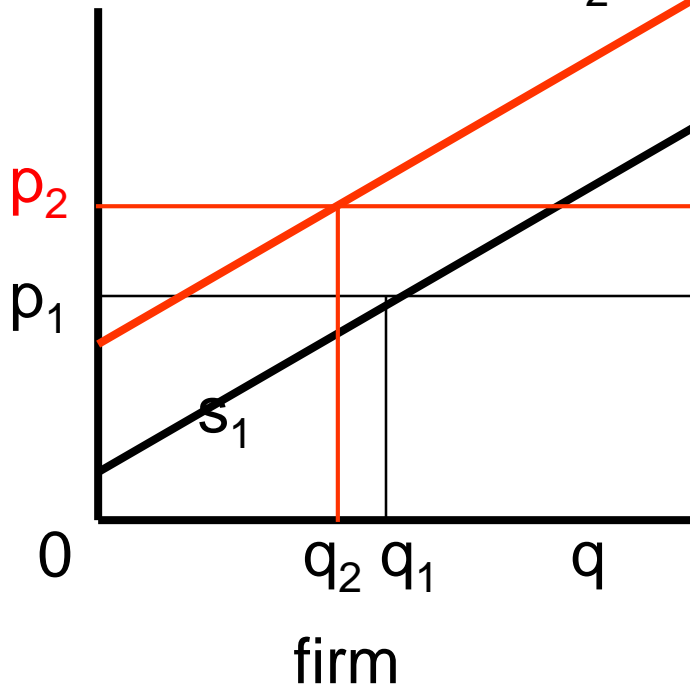
Increase in input prices  $\uparrow$  MC.

Supply curves  $s_1$  &  $S_2$  move up to  $s_2$  &  $S_2$ .

Industry diagram price  $\uparrow$  from  $p_1$  to  $p_2$ ,  $Q \downarrow$  from  $Q_1$  to  $Q_2$ .

Firm diagram firm quantity  $\downarrow$  from  $q_1$  to  $q_2$ .

# Change in marginal cost



Increase in input prices  $\uparrow$  MC.

Supply curves  $s_1$  &  $S_2$  move up to  $s_2$  &  $S_2$ .

Industry diagram price  $\uparrow$  from  $p_1$  to  $p_2$ ,  $Q$   $\downarrow$  from  $Q_1$  to  $Q_2$ .

Firm diagram firm quantity  $\downarrow$  from  $q_1$  to  $q_2$ .

# Special Case CRS

Usually AC & MC depend on  $q$  and input prices.

With constant returns to scale  $AC = MC$  and does not vary with  $q$ . AC & MC do depend on input prices.

$c$

$$MC = AC = c$$

the firm's  
supply

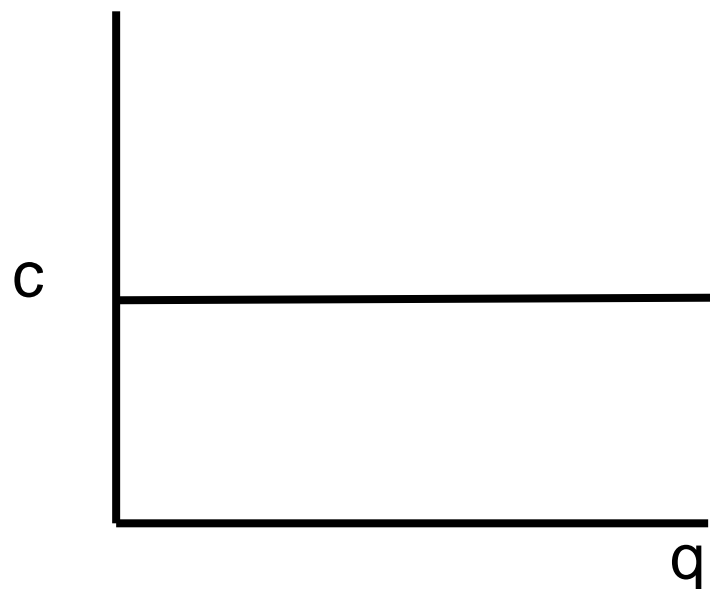
0

$q$

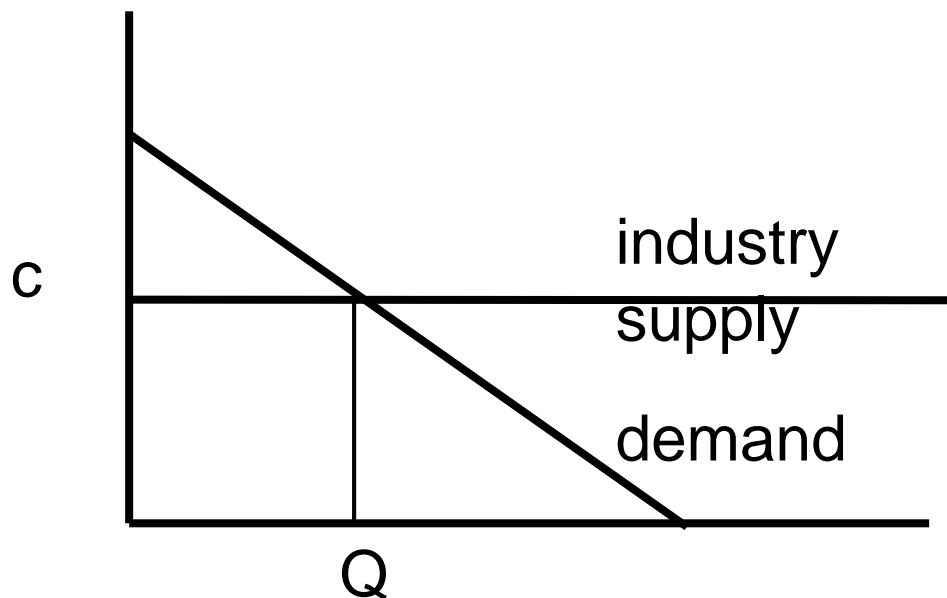
When  $p = c$  the firm makes the same profits (0) at any  $q$ . The firm's output is not determined.

[See previous lecture](#)

# Equilibrium in a market with a fixed set of price taking firms and CRS



firm



industry

industry supply = horizontal sum of firms' supply

industry quantity  $Q$  comes from intersection supply and demand on the industry diagram.

Price =  $c$ . Firm supply is not determined.

# Firms with different costs & economic rent

### 3. Firms with different costs & economic rent

Up to now:

models in which all firms have including potential entrants have the same costs.

Different firms may have different costs, due to different technologies or access to different quality inputs.

I will look at the effects of access to different quality inputs.



# Case 1: Entry Costs

There is a cost to entering the industry. For firms already in the industry this is a sunk cost and not an opportunity cost.

For potential entrants this is an opportunity cost.

If at the industry price

$0 < \text{profits for firms in the industry} < \text{entry costs}$

firms in the industry make positive profits in entry and exit equilibrium.

## Case 2: some firms have higher quality inputs which can't be traded

Some inputs come in different qualities.

If either the difference can't be observed so high and low quality inputs trade at the same price

or it is not possible to trade the inputs

firms with higher quality inputs have lower costs and higher profits.

# Case 3: some firms have higher quality inputs which can be traded.

Some inputs come in different qualities.

Now suppose these inputs can be traded.

Higher quality inputs have higher prices.

In the simplest case all the extra profits that firms have with high quality inputs that can't be traded go into the price of these inputs.

Now firms with high quality inputs make 0 profits.

# Economic Rent

Originally this story about different quality inputs was about farming with varying land quality.

With no market for land farmers with high quality land make economic profits.

With a market for land all the profits go to the landowner in the form of rent.

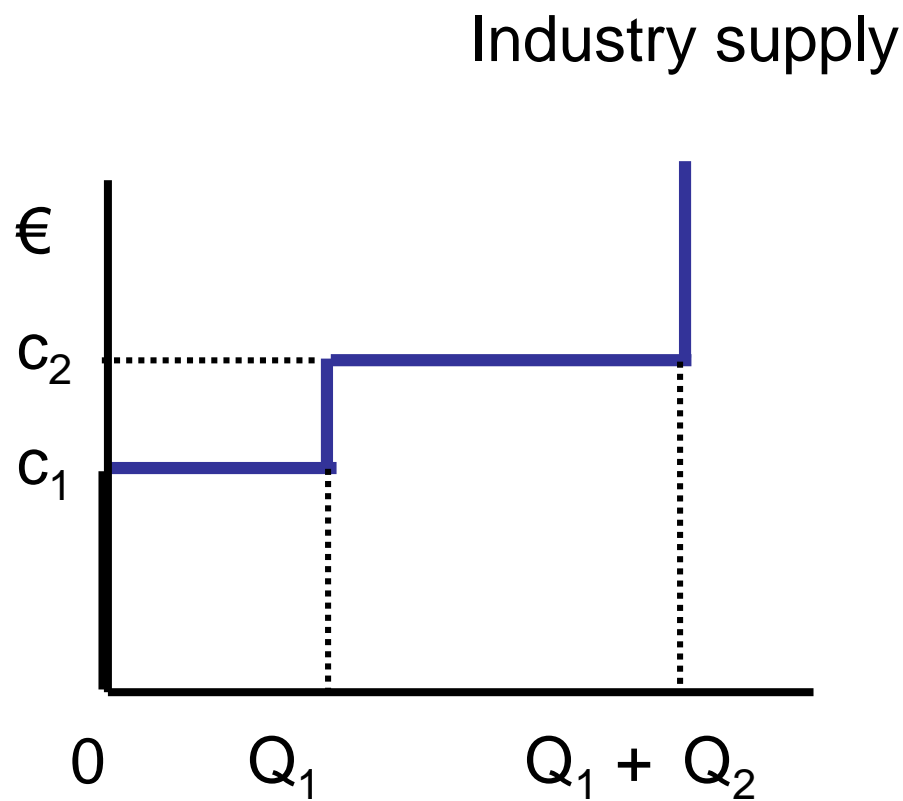
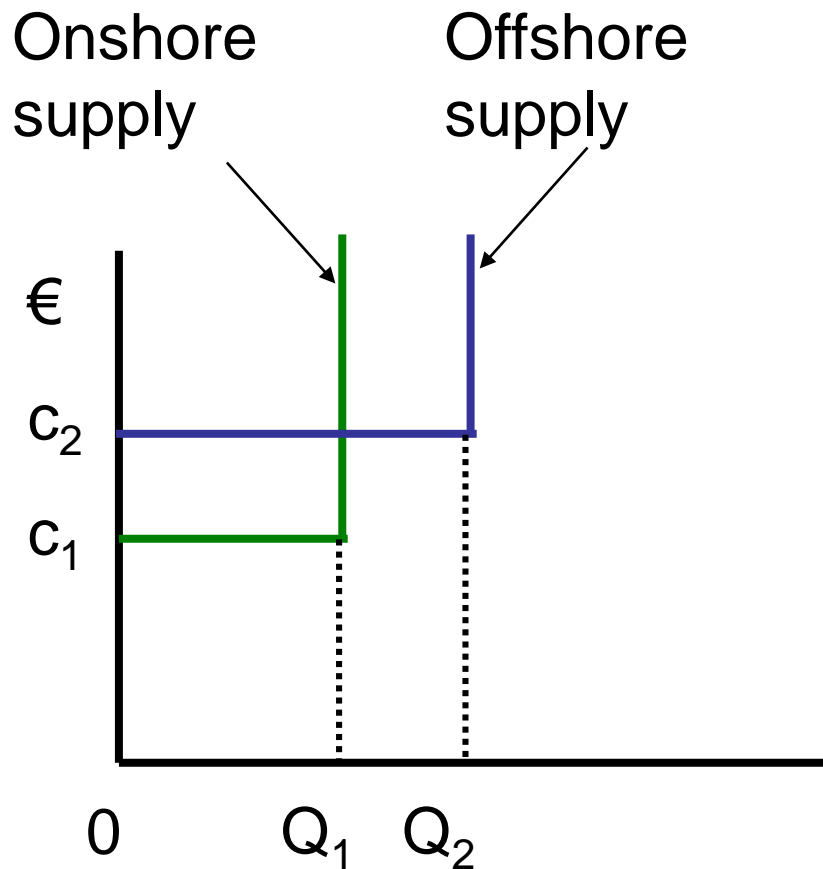
Farmers that do not own the land make 0 profits.

Rent is sometimes used as a term for profits.

# A very simple model of rent in a price taking oil industry

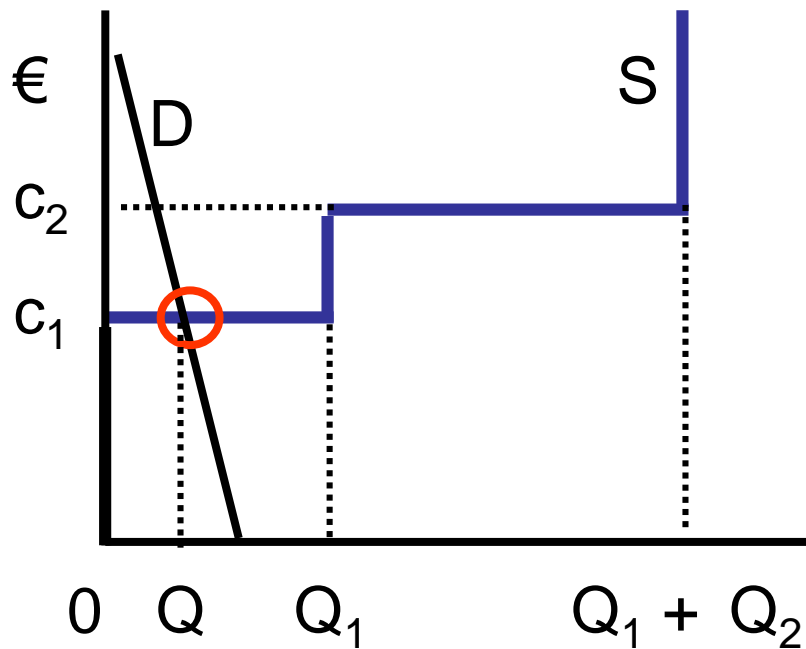
- There are two types of input.
- “High quality” onshore wells,  $MC = AC = c_1$ ,  
total capacity  $Q_1$ .
- “Low quality” offshore wells,  $MC = AC = c_2 > c_1$ ,  
total capacity  $Q_2$ .





add supply curves horizontally

# Industry supply & demand with different demand curves



Solve for  $S = D$

accurate diagram

or find  $Q$  at  $p = c_1$  &  $p = c_2$

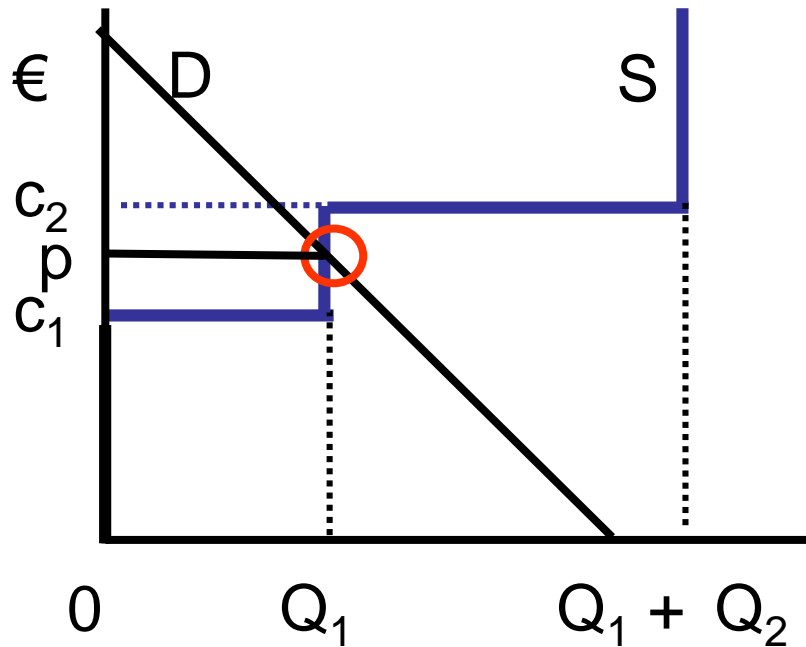
then use a diagram

Supply = demand at  $Q$  where  $0 < Q < Q_1$ . Then  $p = c_1$ .

Onshore wells make 0 profits & produce less than maximum  $Q_1$ .

Offshore wells do not produce and make 0 profits.

# Industry supply & demand with different demand curves



Solve for  $S = D$

accurate diagram

or find  $Q$  at  $p = c_1$  &  $p = c_2$

then use a diagram

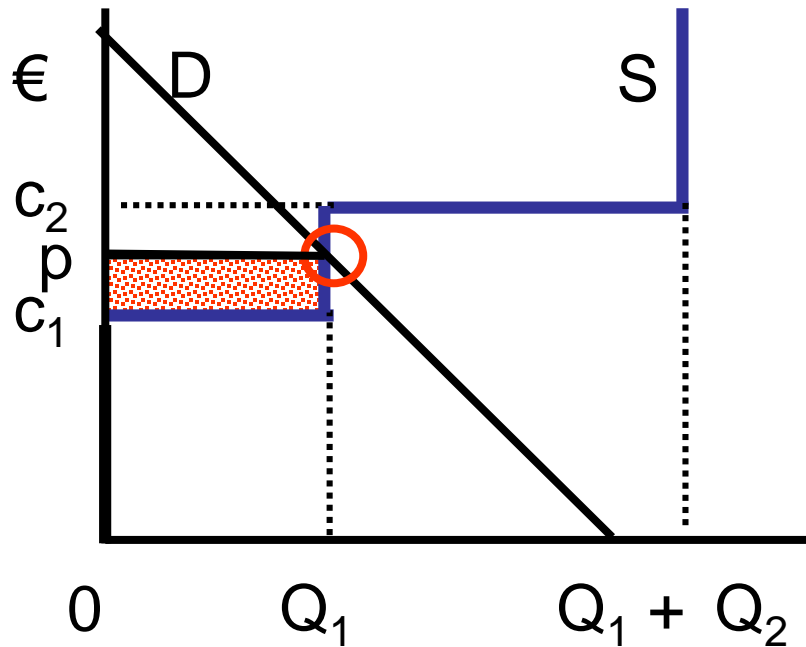
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# Industry supply & demand with different demand curves



Solve for  $S = D$

accurate diagram

or find  $Q$  at  $p = c_1$  &  $p = c_2$

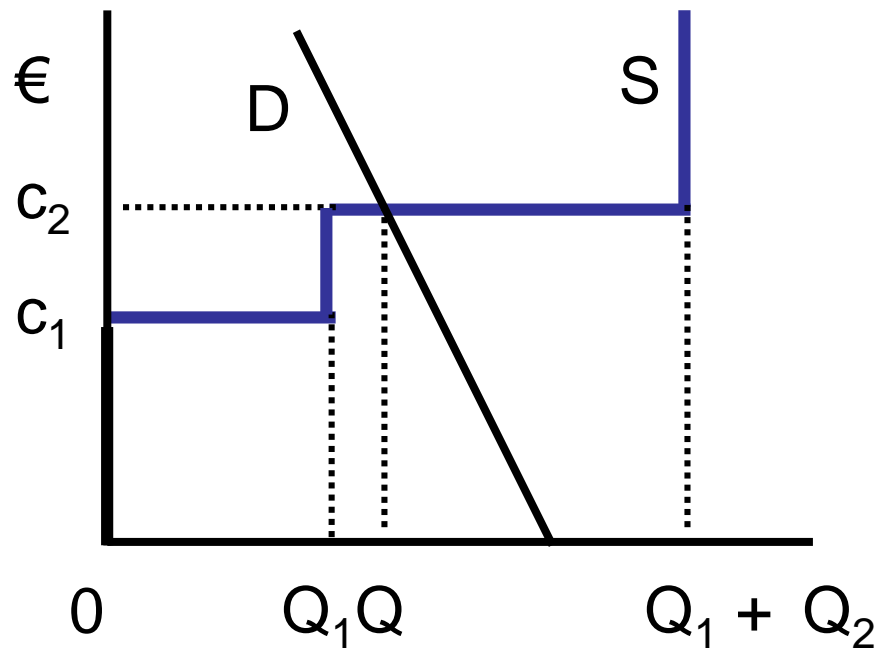
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# Industry supply & demand with different demand curves

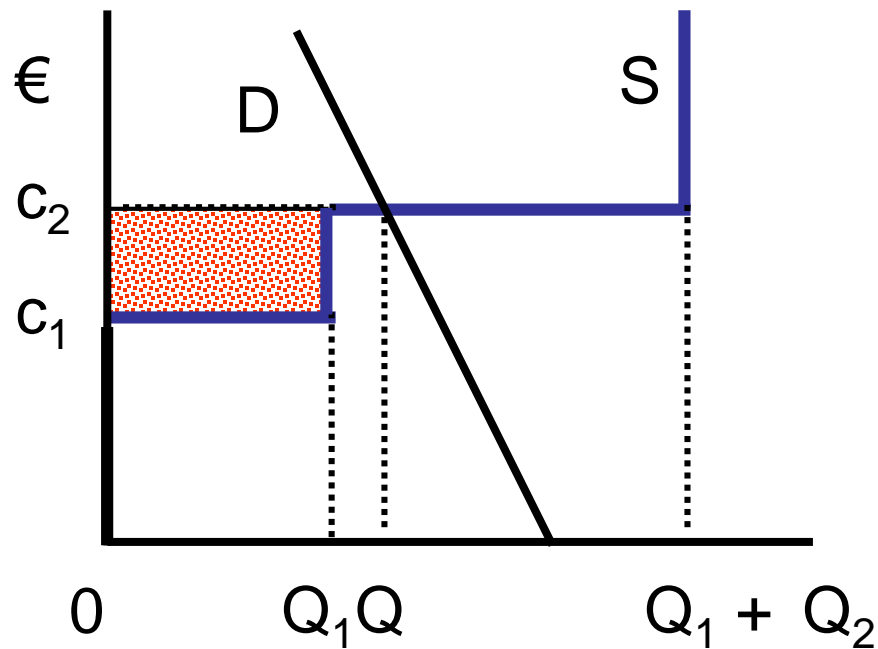


Supply = demand at  $Q$  where  $Q_1 < Q < Q_1 + Q_2$ .  $p = c_2$ .

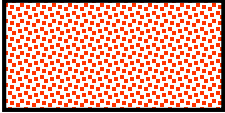
Onshore wells make profits & produce the maximum  $Q_1$

Offshore wells produce  $< Q_2$  & make 0 profits

# Industry supply & demand with different demand curves

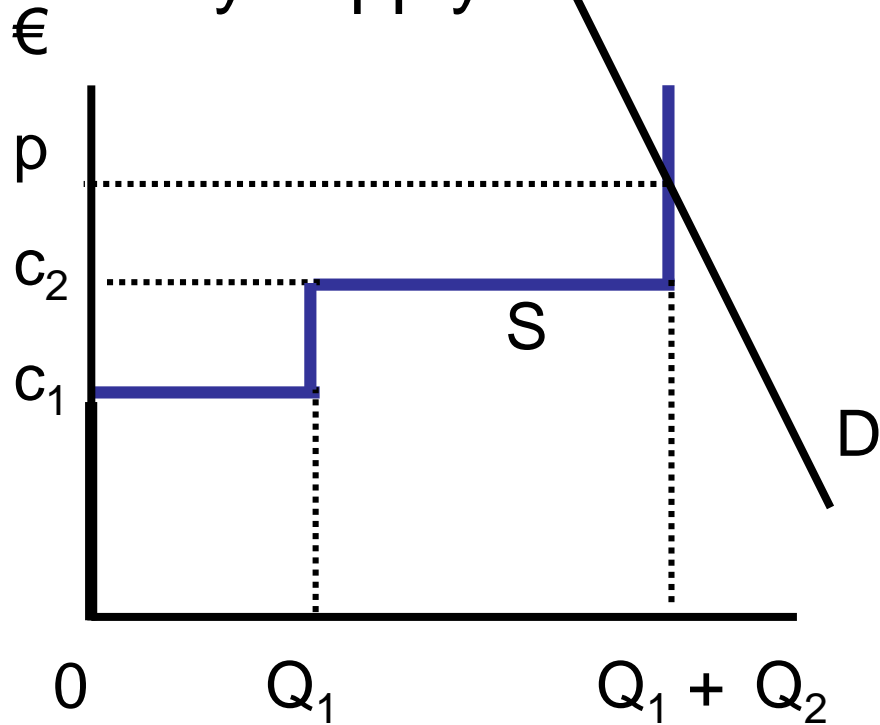


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# Industry supply & demand with different demand curves

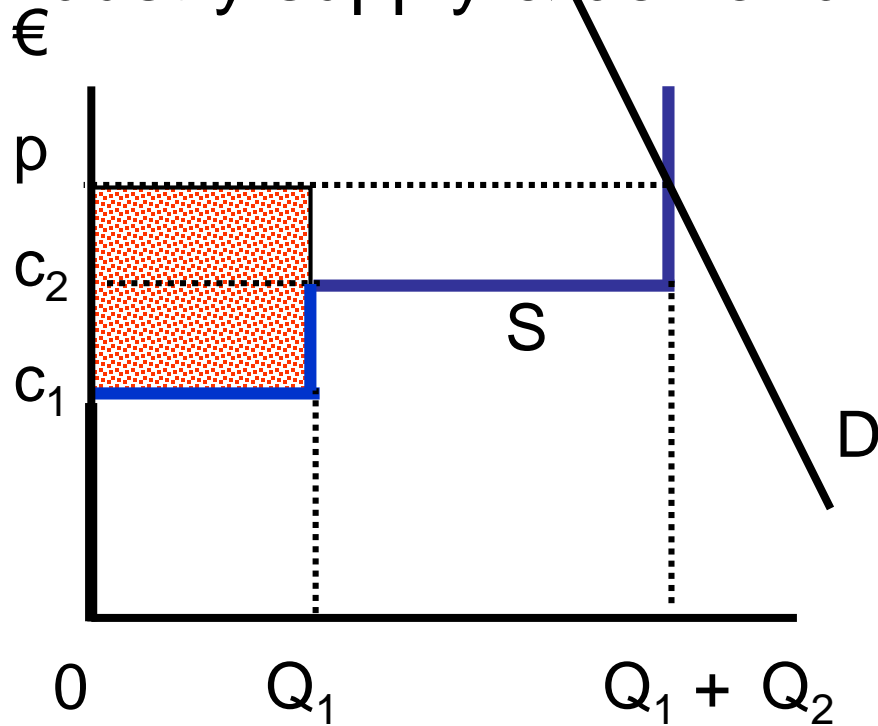


Supply = demand at  $Q = Q_1 + Q_2$ .  $p > c_2$ .

Onshore wells make profits & produce the maximum  $Q_1$


Offshore wells produce make profits & produce the maximum  $Q_2$ .

# Industry supply & demand with different demand curves

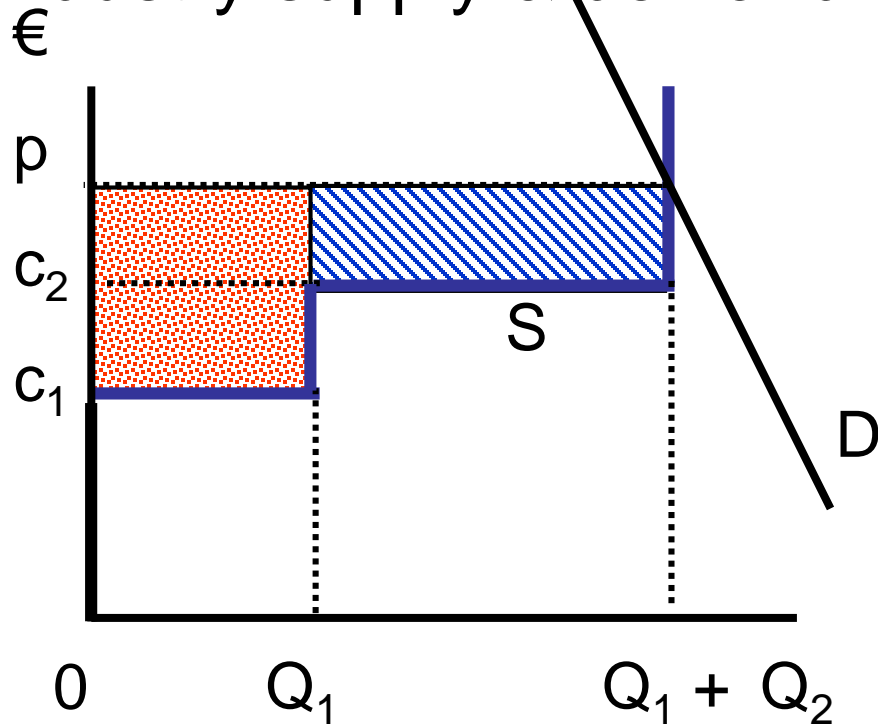


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# Industry supply & demand with different demand curves



Supply = demand at  $Q = Q_1 + Q_2$ .  $p > c_2$ .

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# Welfare economics of a tax with supply and demand

## 4. Welfare economics of a tax with supply and demand

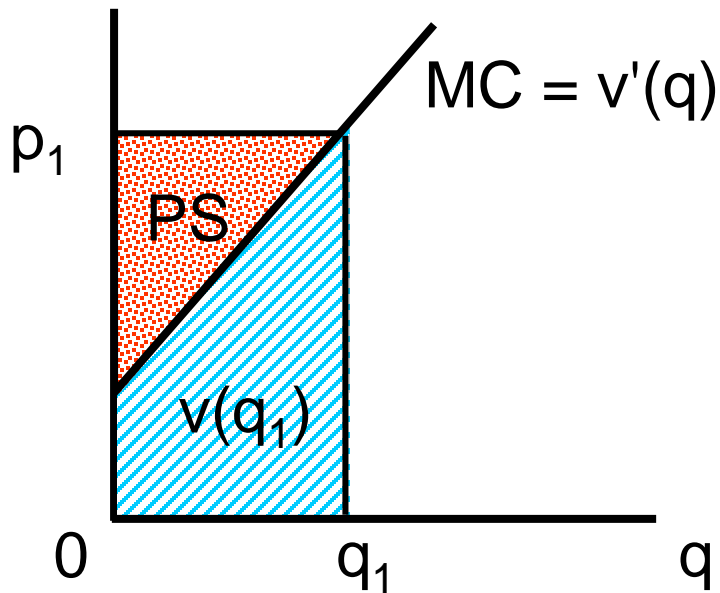
Assume that firms' costs = social costs

A strong assumption requiring no externalities and perfectly competitive input markets

This assumes that distribution is not an issue, the social gain of giving €1 to someone does not depend on wealth, income ....



# Producer surplus for a firm

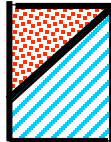


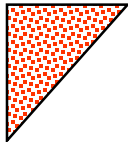
Total cost =  $F + v(q)$   
 = fixed cost + variable cost.

$v'(q)$  = derivative of variable cost  
 = MC.

Integration,  $v(q_1)$  is area under MC curve.

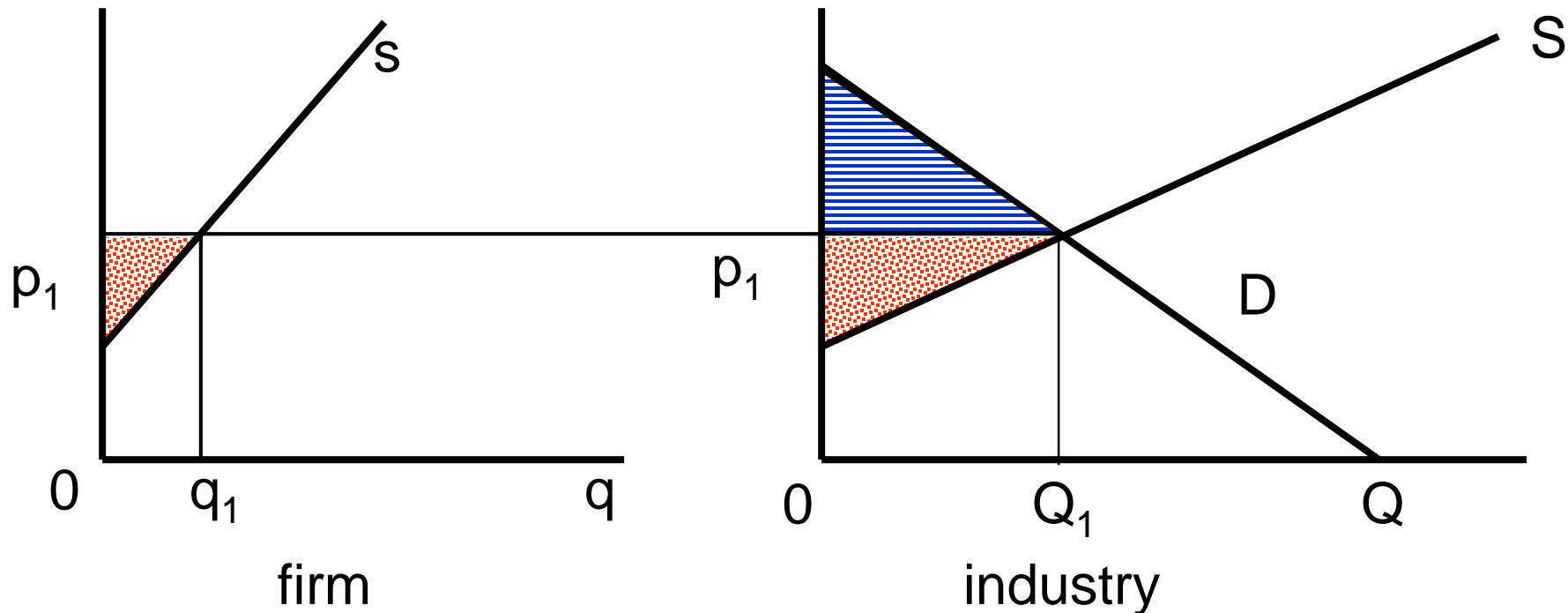


Total revenue =  $p_1 q_1 =$  

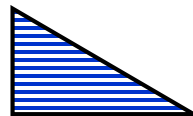
**Producer surplus**  = total revenue – variable cost

If there is no fixed cost producer surplus = profit.

# Industry consumer and producer surplus

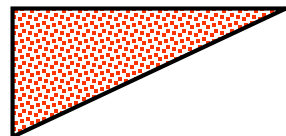


industry consumer surplus



sum the firm's supply curve horizontally to get industry supply

and industry producer surplus



# Welfare economics with supply and demand example 1 tax

Assume for simplicity the tax  $t$  is per unit sold, not % of price

Example excise tax on cigarettes, alcohol, petrol.

The tax  $t$  per unit increases marginal cost by  $t$

# Effects of a tax in an industry with CRS

If the tax is  $t$  and price paid by consumers is  $p$

firms get  $p - t$ .

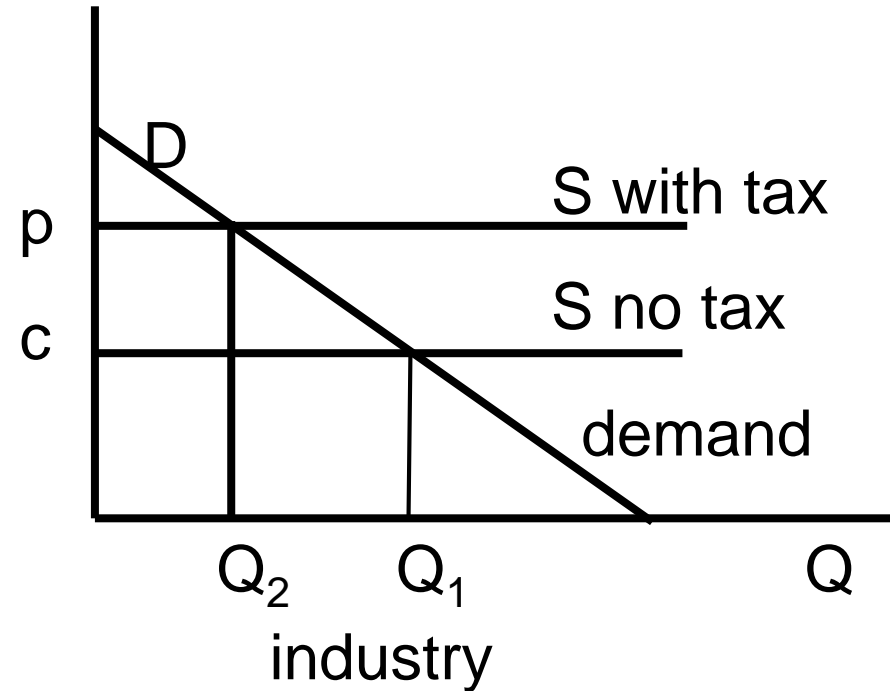
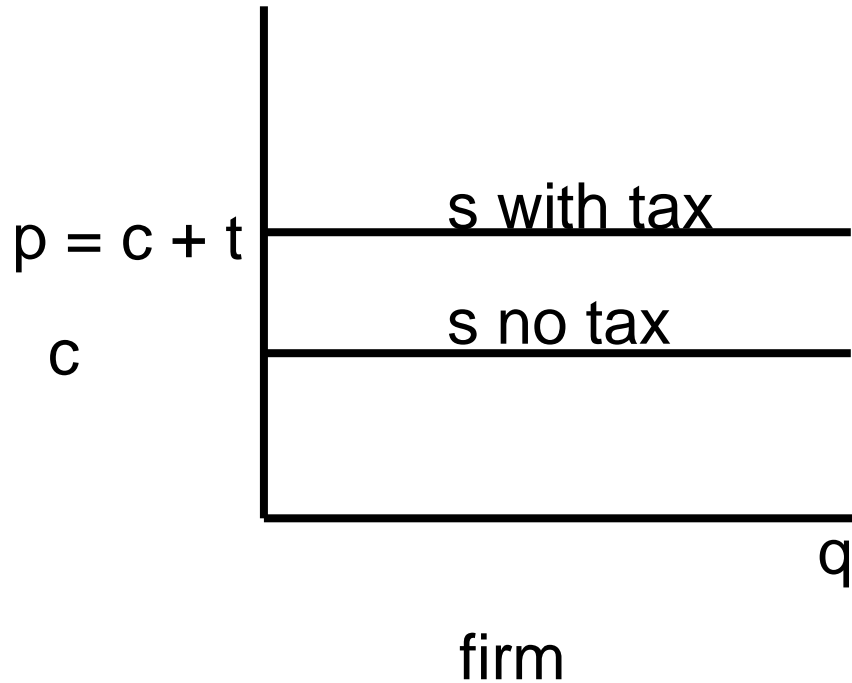
If all firms have CRS so constant AC & MC  $c$   
profits from supplying  $q$  are  $(p - t - c)q$

profits are 0 when  $p = c + t$ ,

supply with tax is perfectly elastic at  $c + t$

Assuming that input prices don't change when the industry expands industry supply with the tax is horizontal at  $c + t$ .

# Effect of an excise tax in an industry with CR

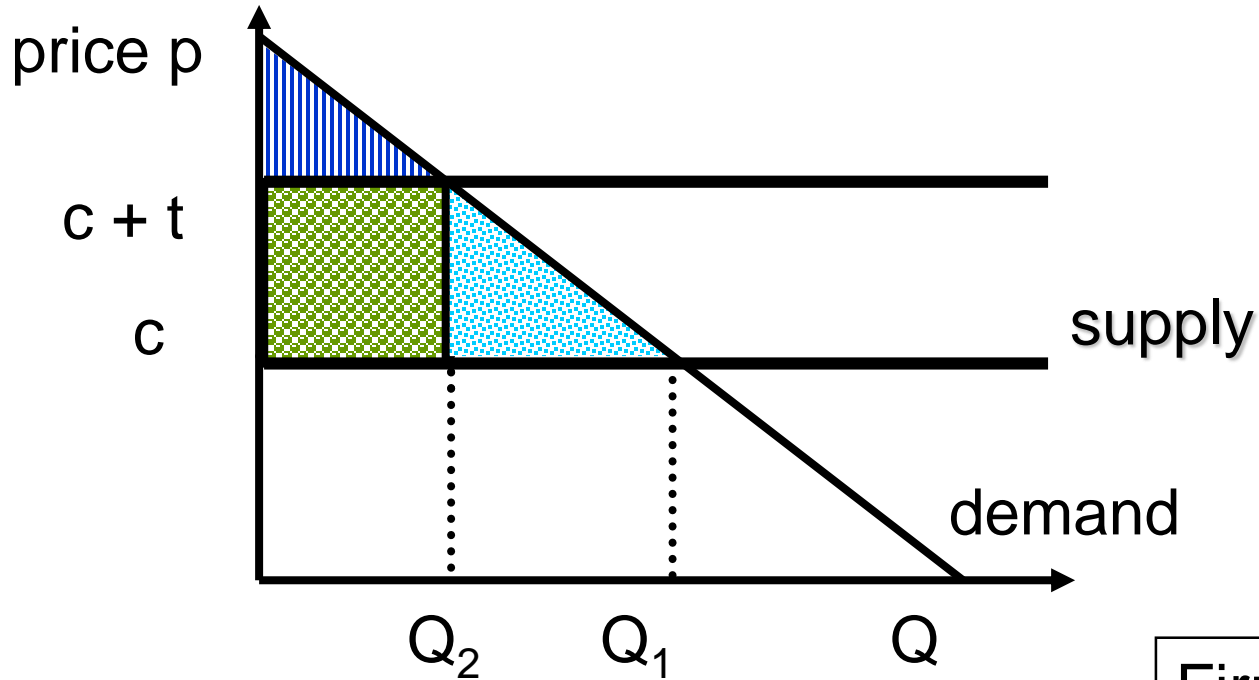


with no tax price is  $c$  ( $= AC = MC$ ), industry produces  $Q_1$

with the tax price is  $c + t$ , industry produces  $Q_2$ ,

tax revenue  $= tQ_2$

# Effects of an excise tax with CRS



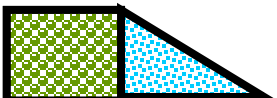
Firms make 0 profits with and without tax.

With CRS there is no producer surplus.

With no tax price =  $c$ , industry output  $Q_1$

Consumer surplus = entire shaded area.

With tax  $t$  price =  $c + t$ ,  $Q$  falls to  $Q_2$ , tax revenue = 

loss consumer surplus =  > tax revenue

# Definition of deadweight loss due to tax

**deadweight loss** =

loss of producer & consumer surplus – tax revenue

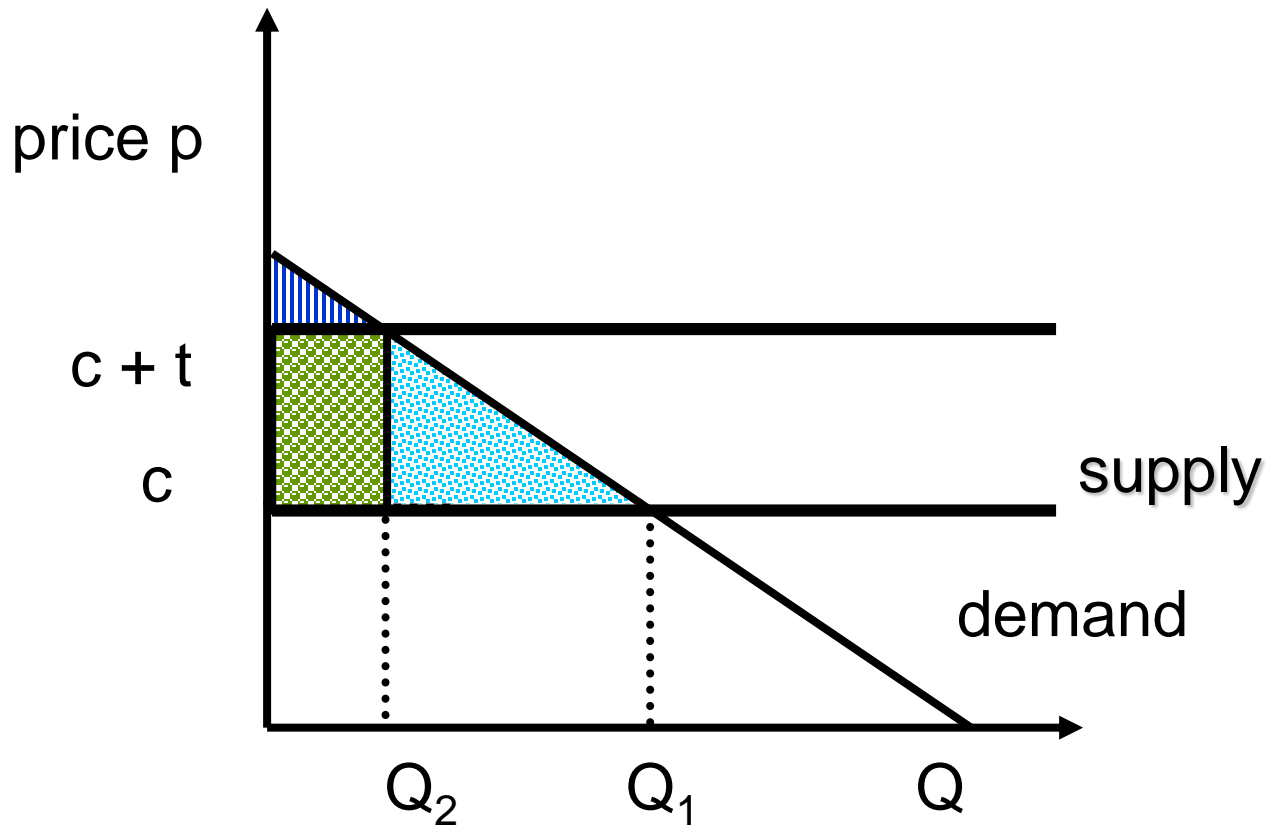
In this case loss of producer surplus = 0

loss of consumer surplus  $\approx$  equivalent variation

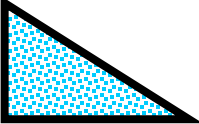
deadweight loss  $\approx$  equivalent variation – tax revenue

= excess burden of the tax to the consumer

# Effects of an excise tax with CRS



Here demand is more elastic,

deadweight loss area  =  $\frac{1}{2} t (Q_1 - Q_2)$

is a larger fraction of tax revenue area  =  $t Q_2$



$$\text{Deadweight loss} = -\frac{1}{2}t\Delta Q$$

(- sign because  $\Delta Q = Q_2 - Q_1 < 0$ )

Economists always want formulae in terms of elasticity

$$\Delta Q = \left( \frac{\Delta Q}{\Delta p} \frac{p}{Q} \right) \left( \frac{\Delta p Q}{p} \right) = e \frac{\Delta p Q}{p} = e \frac{tQ}{p}$$

where  $e = \text{elasticity} = \frac{\Delta Q}{\Delta p} \frac{p}{Q}$  so with downward sloping

demand curves elasticity is negative.

$$\text{Deadweight loss} = -\frac{1}{2}t\Delta Q = -\frac{1}{2}te \frac{tQ}{p} = \frac{1}{2}(-e)(tQ) \left( \frac{t}{p} \right)$$

$$\text{Deadweight loss} = -\frac{1}{2}te\frac{tQ}{p} = \frac{1}{2}(-e)(tQ)\left(\frac{t}{p}\right)$$

$tQ$  = tax revenue,

$t/p$  = proportionate increase in price

$$\frac{\text{deadweight loss}}{\text{tax revenue}} = \frac{1}{2}(-\text{elasticity})(\text{proportionate increase in price})$$

## Implication

Given target **tax revenue** it is better to raise tax revenue by taxing goods whose demand is inelastic.

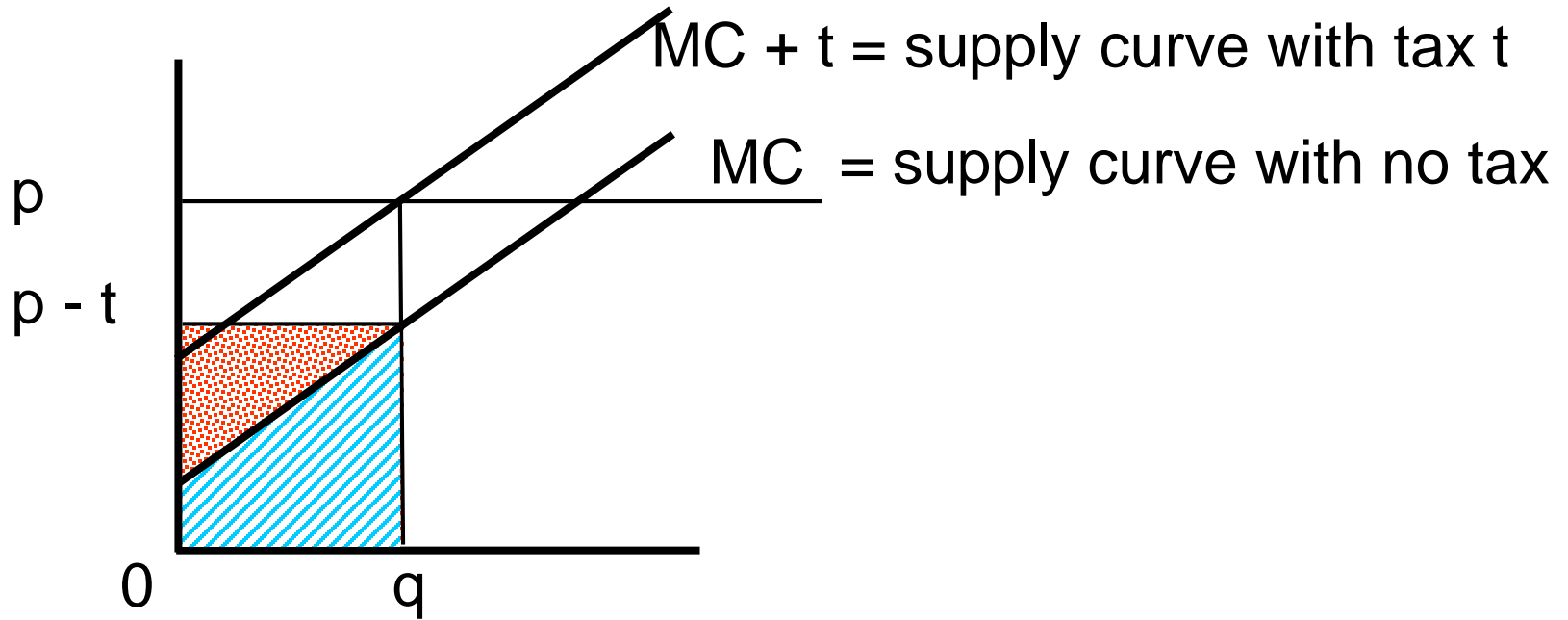
But this ignores distribution.

If the good is a large proportion of the consumption of poor people and you want to take this into account this argument is too simple.

The analysis can be extended to take explicit account of distributional value judgements, but has to be done with maths not diagrams.

Not in this course.

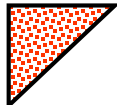
# Profit maximization with an excise tax & increasing MC



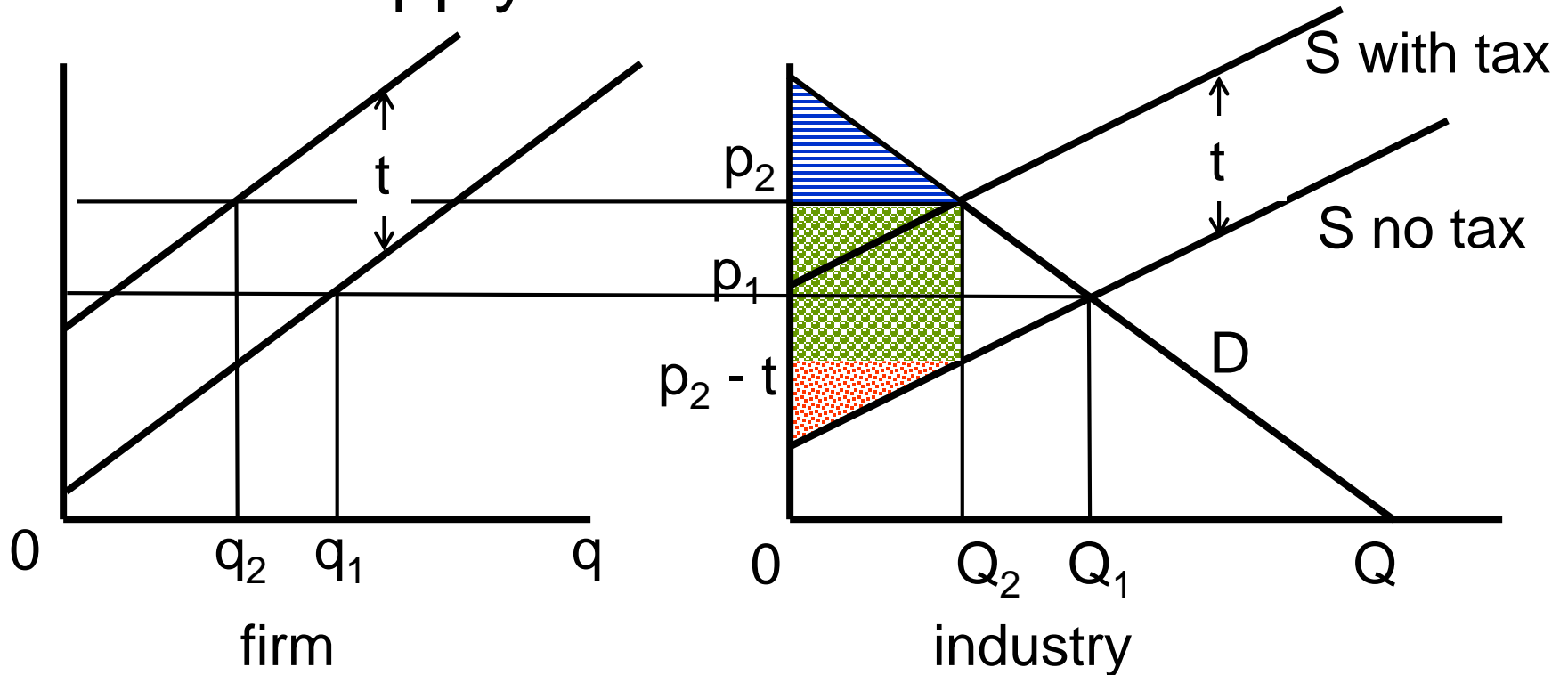
If there is a tax  $t$ , consumers pay  $p$ , firm gets  $p - t$

Profit maximization implies  $p - t = MC$ , or  $p = MC + t$ .

The tax shifts the supply curve upwards by  $t$ .

Producer surplus with tax = 

# Supply and demand with tax

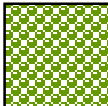


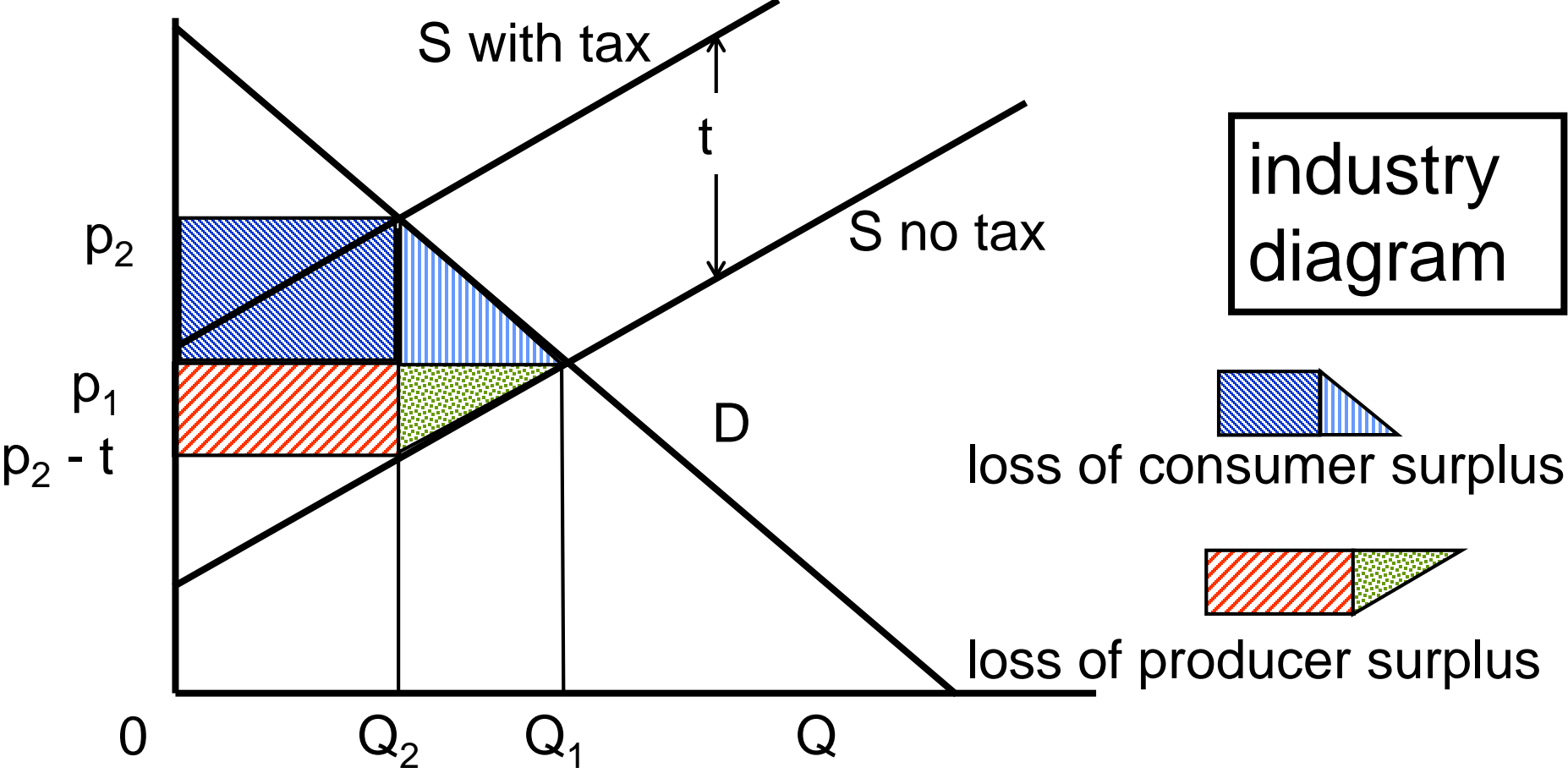
with no tax price  $p_1$ , industry  $Q_1$ , firm  $q_1$

With tax price  $p_2$  industry  $Q_2$  at intersection  $D$  and  $S$  with tax.

consumer surplus = 

producer surplus = 

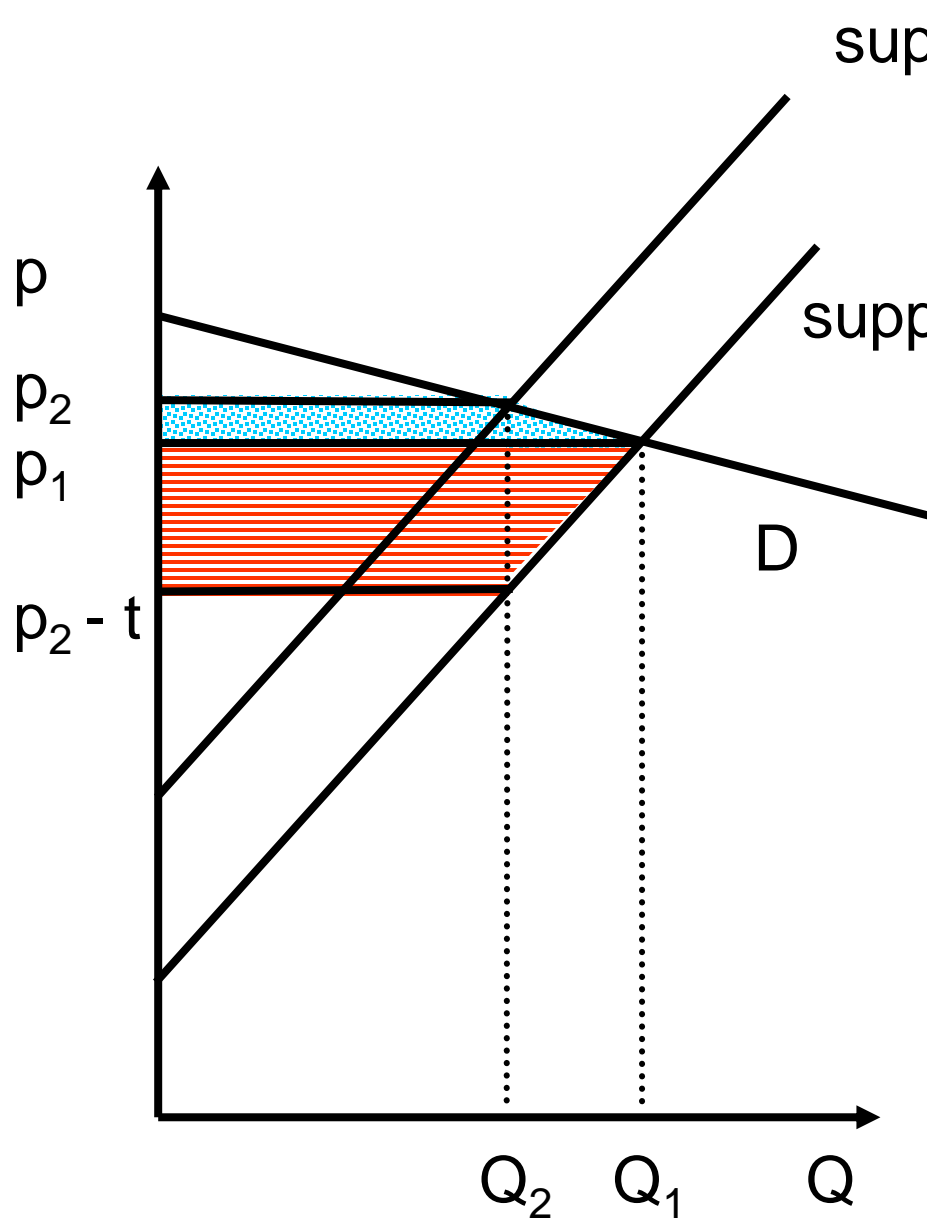
tax revenue  $tQ_2$  = 



deadweight loss = loss of surplus – tax revenue

Any form of tax except lump sum taxes involves a deadweight loss.

Politically acceptable lump sum taxes are not feasible.



supply with tax

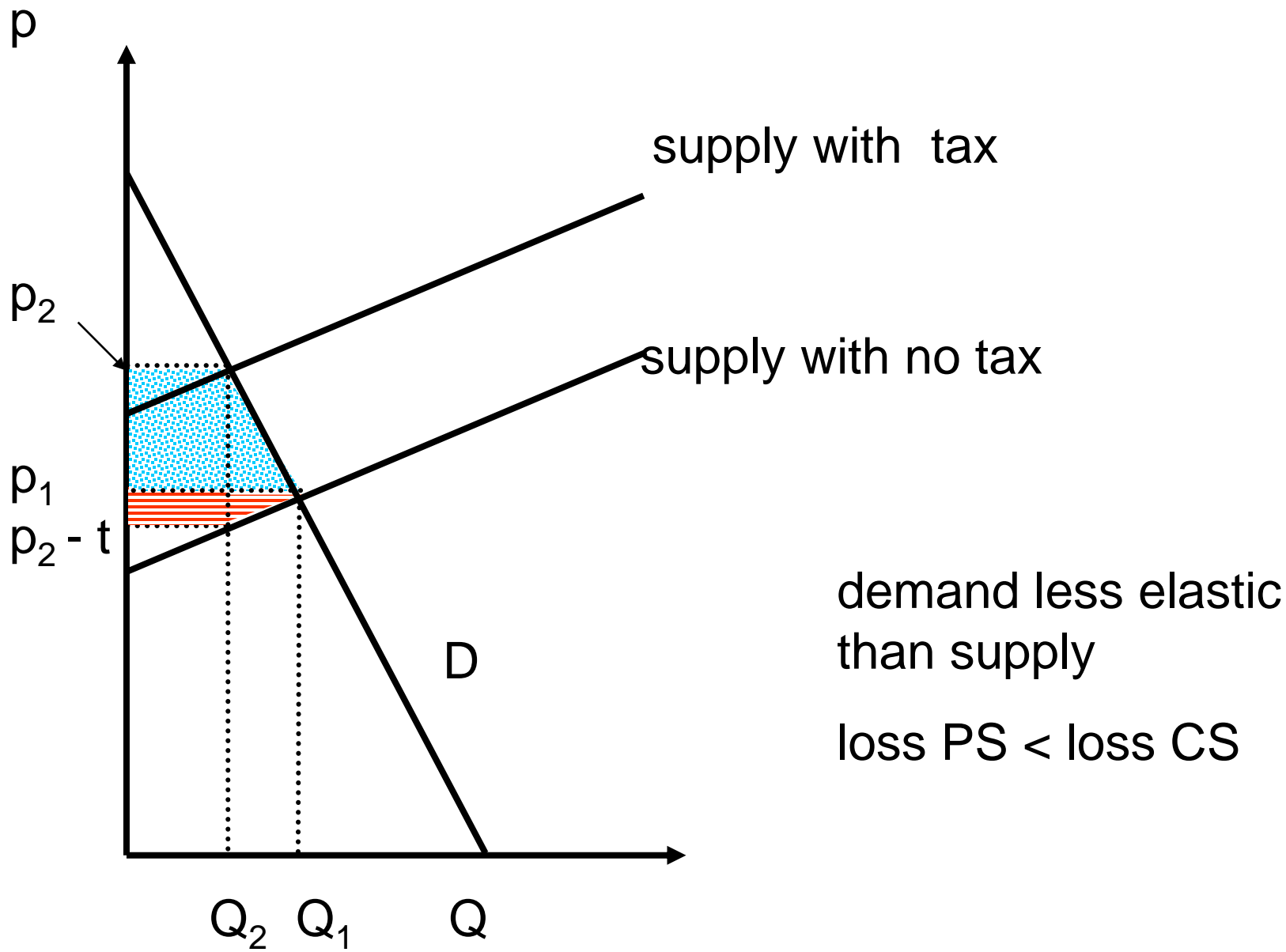
supply with no tax

D

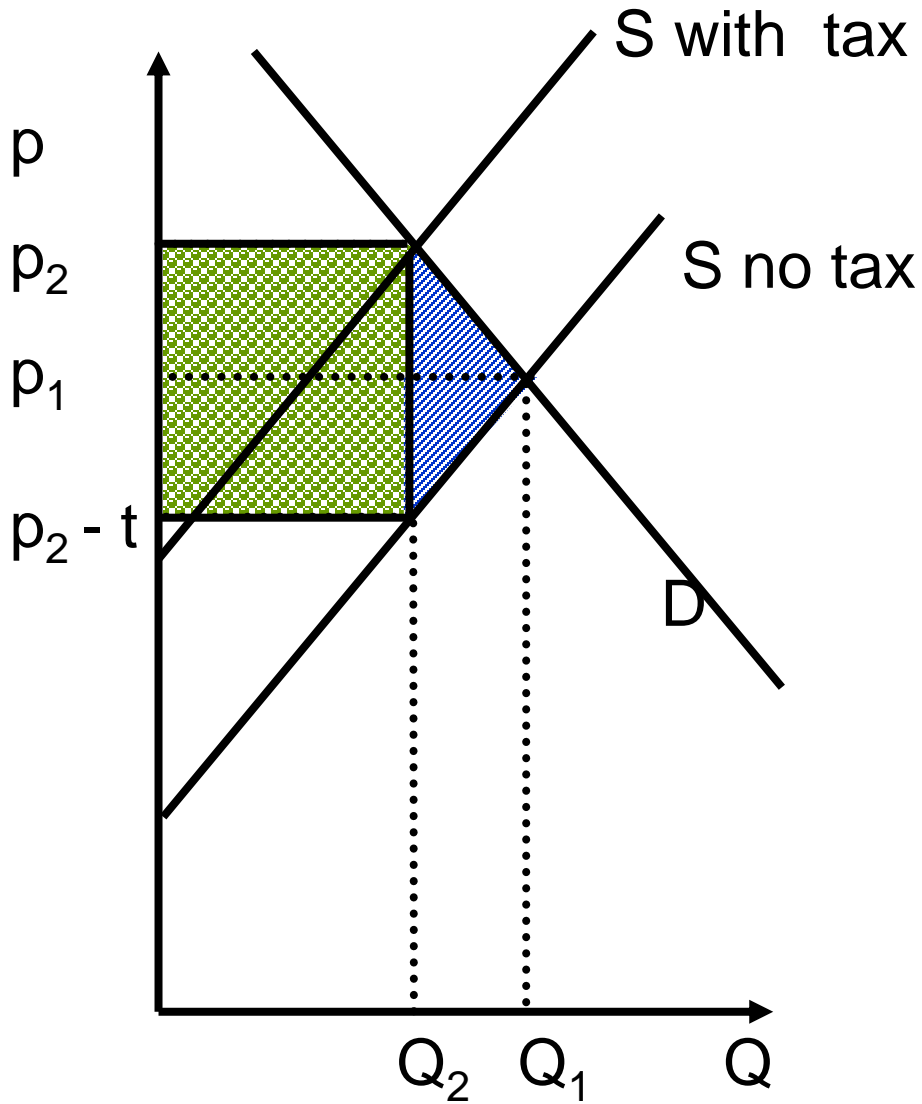
demand more elastic  
than supply

loss PS > loss CS

Price paid by consumer  
increases much less  
than tax.



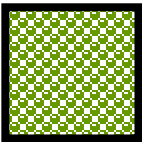


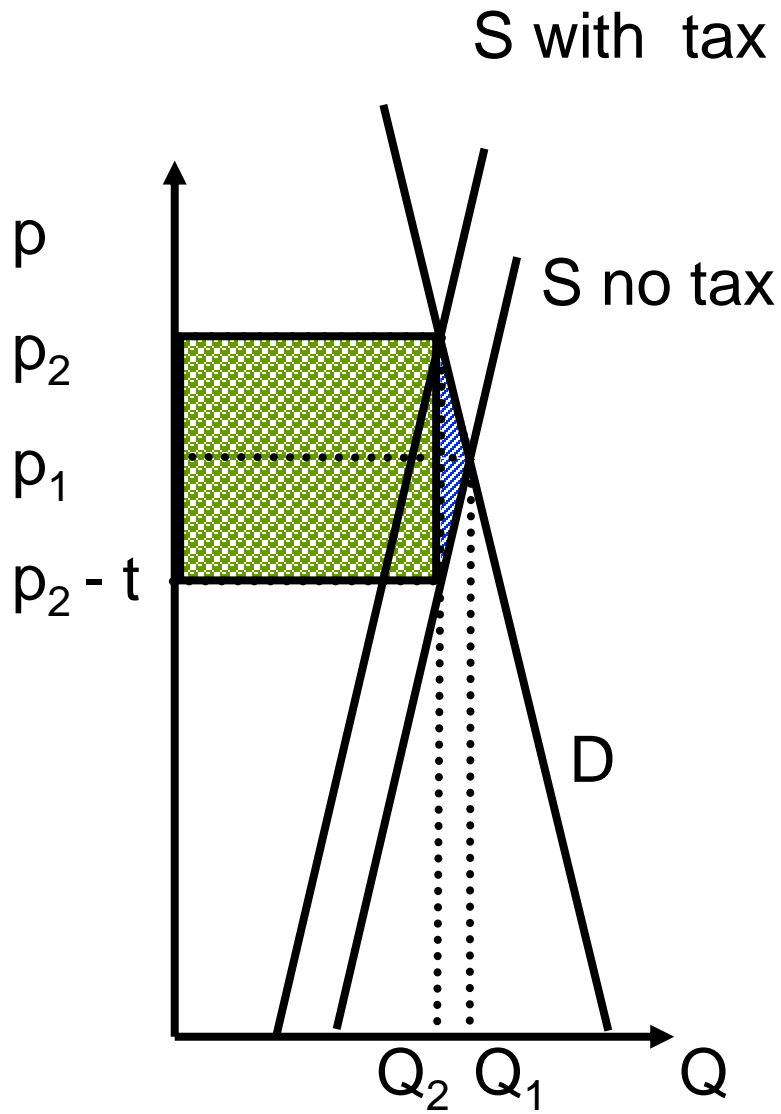


supply and demand are elastic,

deadweight loss is a large fraction

of tax revenue

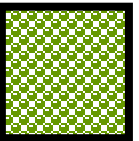




supply and demand are inelastic,

deadweight loss is a small fraction

of tax revenue



# Welfare economics of a subsidy with supply and demand

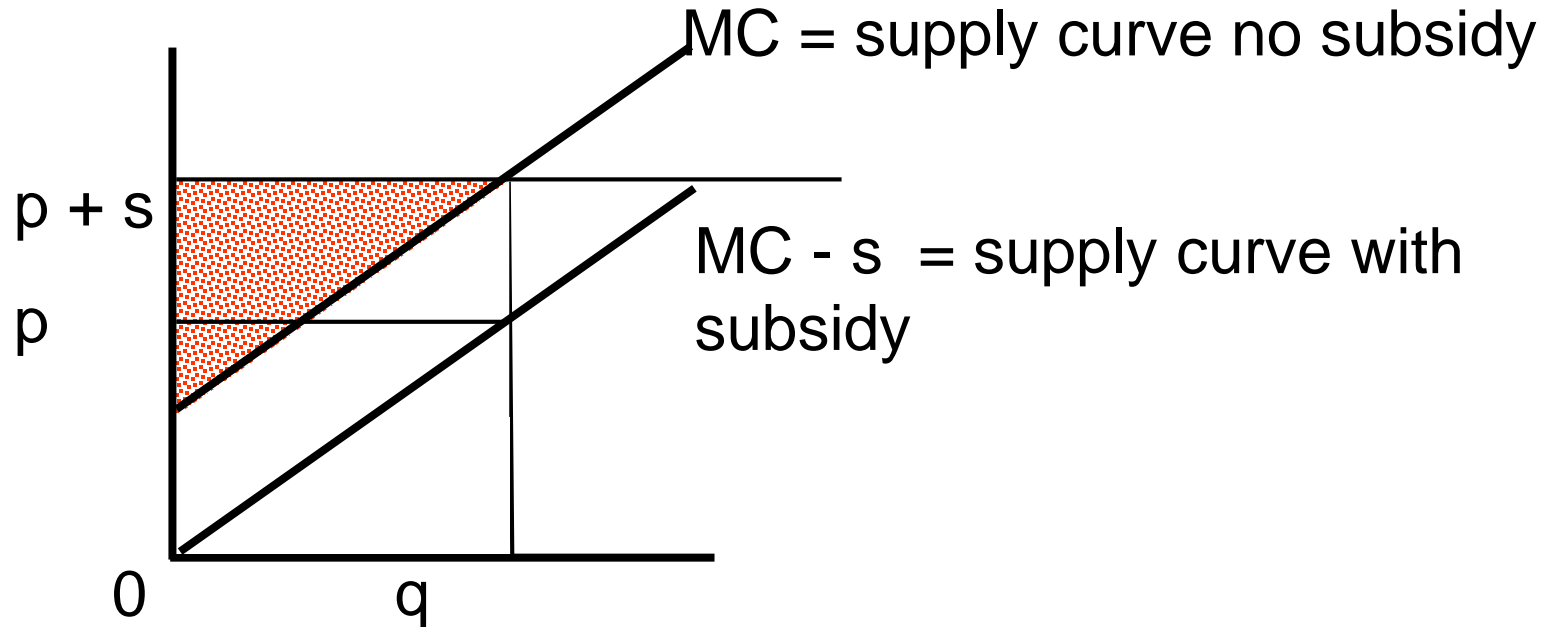
## 5. Welfare economics of a subsidy with supply and demand

Assume for simplicity the subsidy  $s$  is per unit sold, not % of price,

subsidies are particularly common on food, housing and agricultural products.

With a subsidy  $s$  consumers pay  $p$ , producers get  $p + s$  so supply at the point where  $MC = p + s$  or  $p = MC - s$

# Supply by a firm with a subsidy



If there is a subsidy  $s$  & consumers pay  $p$  the firm gets  $p + s$  on each unit.

Profit maximization implies  $p + s = MC$ , or  $p = MC - s$ .

The subsidy shifts the supply curve downwards by  $s$ .

Producer surplus with subsidy = 

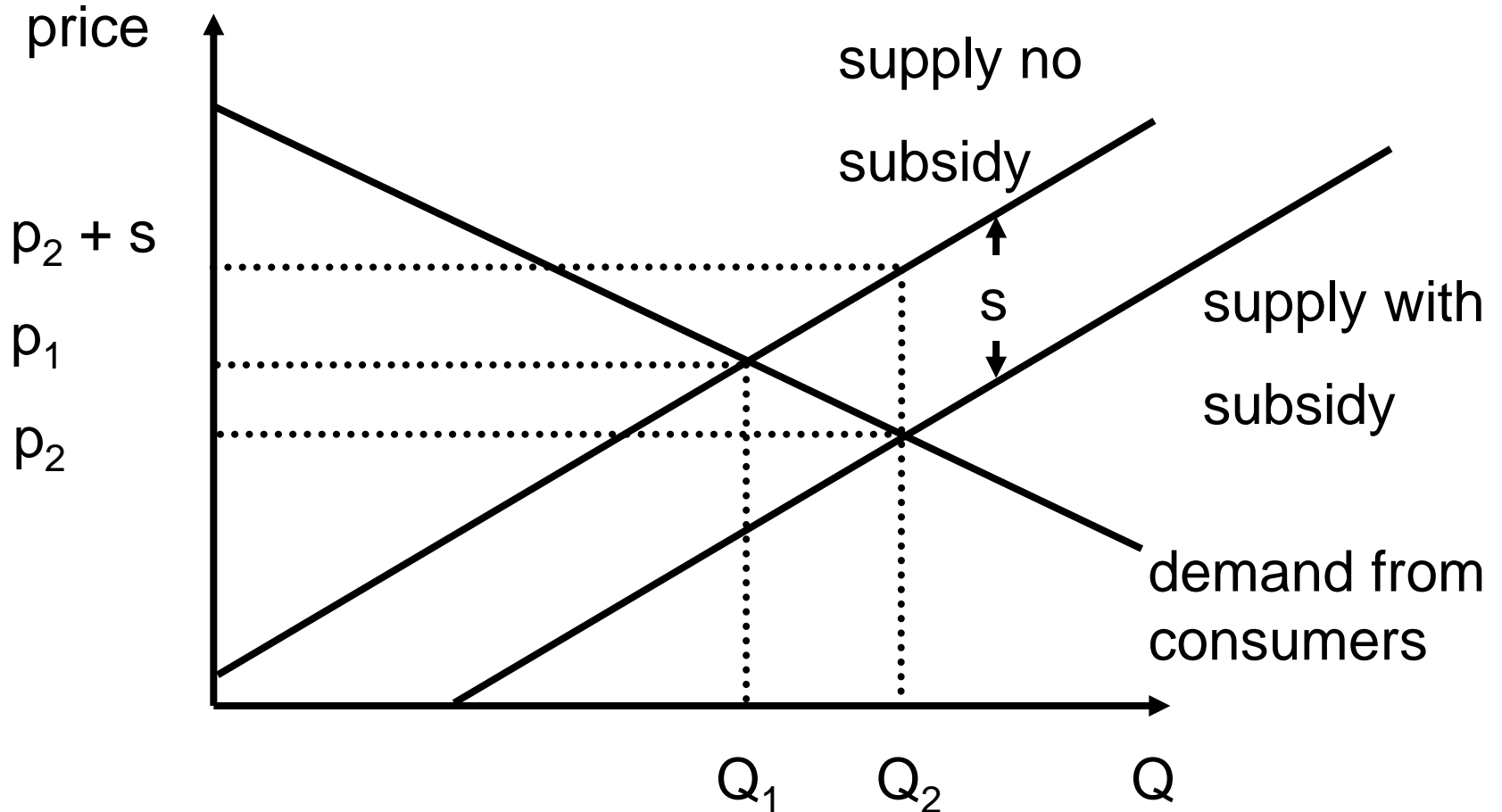
# Supply & demand with a subsidy

With no subsidy  $Q = Q_1$   $p = p_1$ ,


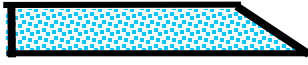
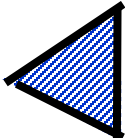
with subsidy price consumers pay falls to  $p_2$

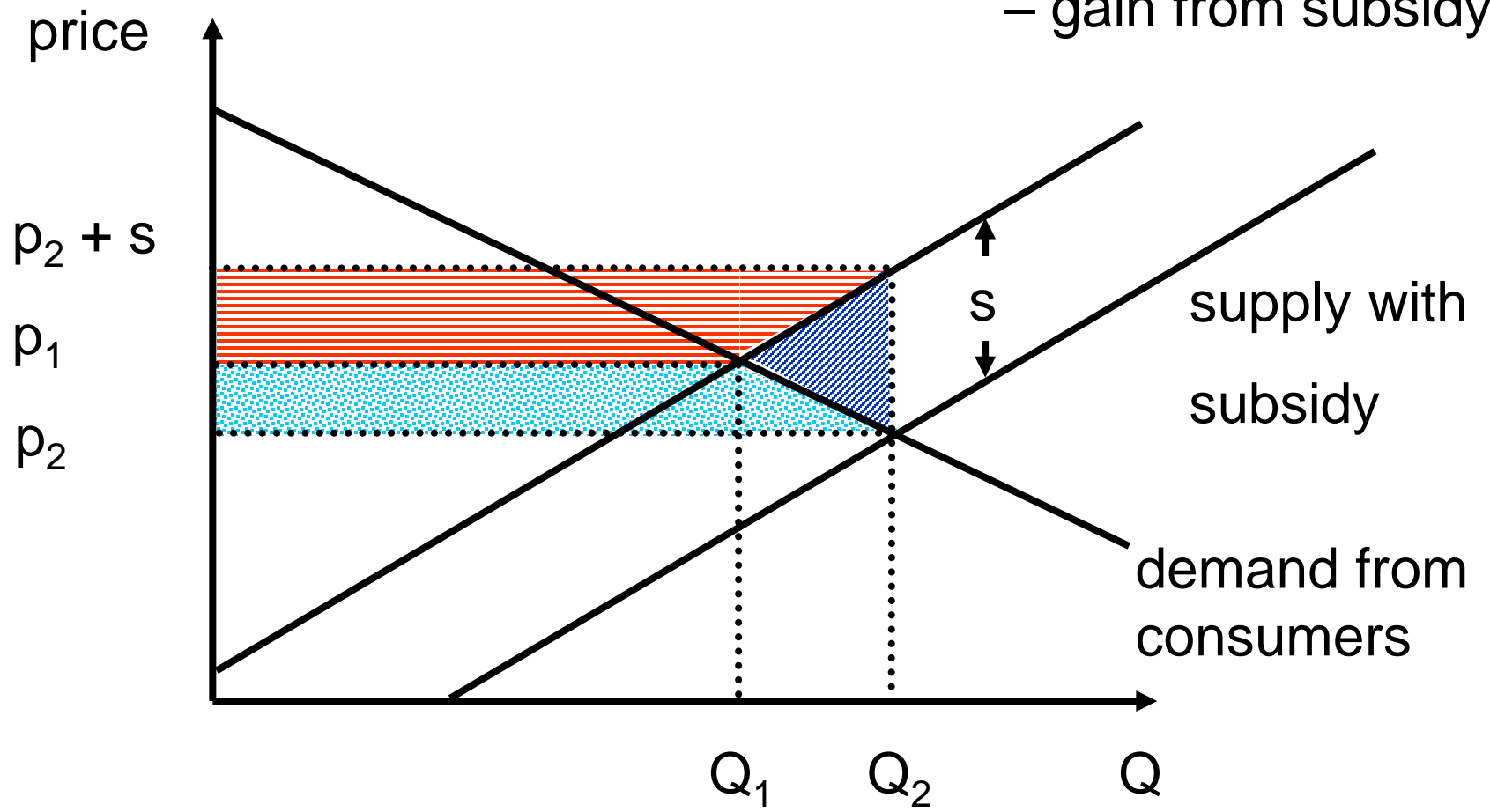
price producers get rises to  $p_2 + s$

$Q$  expands to  $Q_2$





simple welfare economics of a subsidy

- gain in producer surplus 
- gain in consumer surplus 
- entire shaded area = cost of subsidy =  $sQ_2$
- deadweight loss = cost of subsidy 
- gain from subsidy




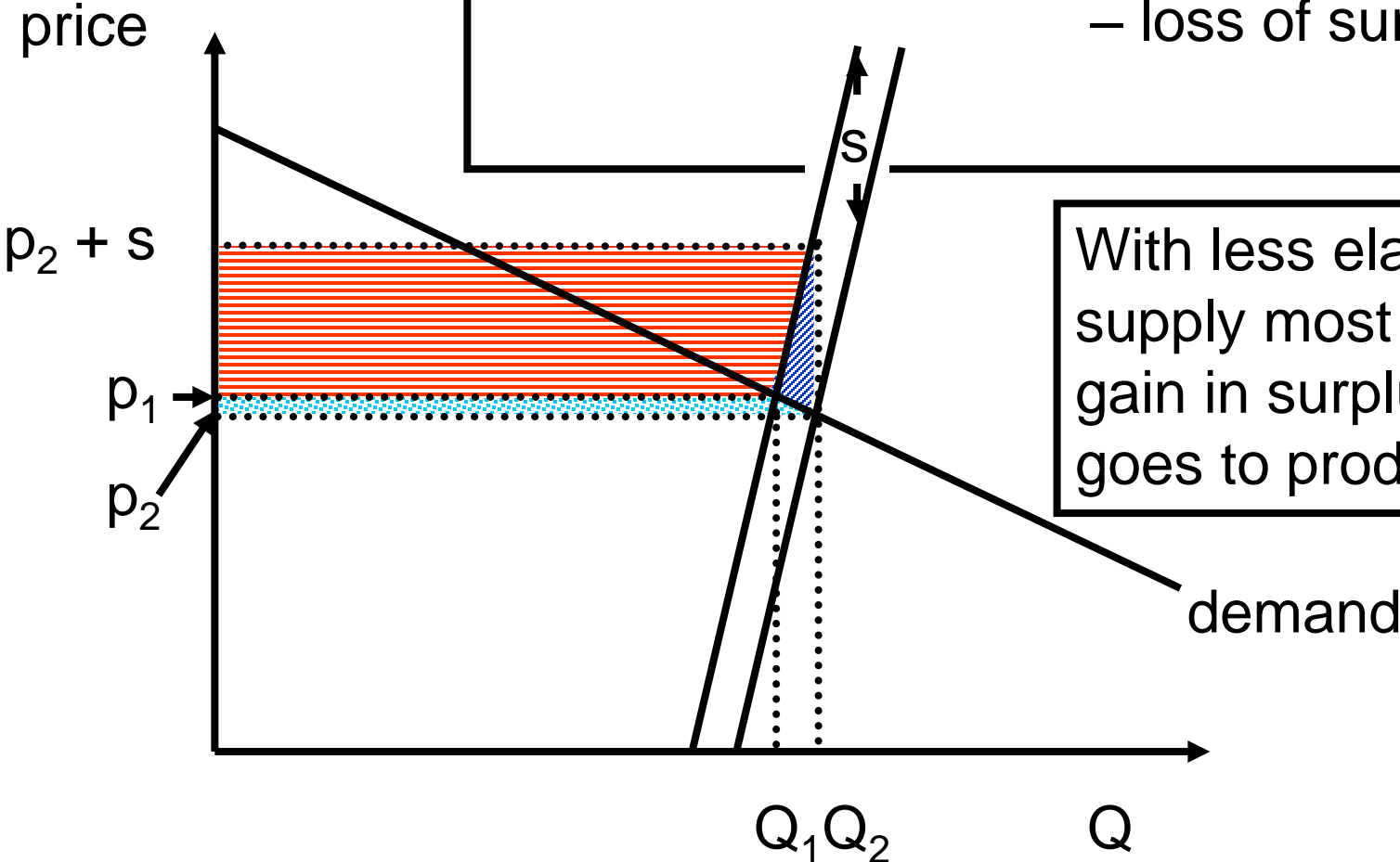
simple welfare economics of a subsidy

gain in producer surplus 

gain in consumer surplus 

entire shaded area = cost of subsidy =  $sQ_2$

deadweight loss = cost of subsidy - loss of surplus 

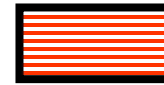


With less elastic supply most of the gain in surplus goes to producers

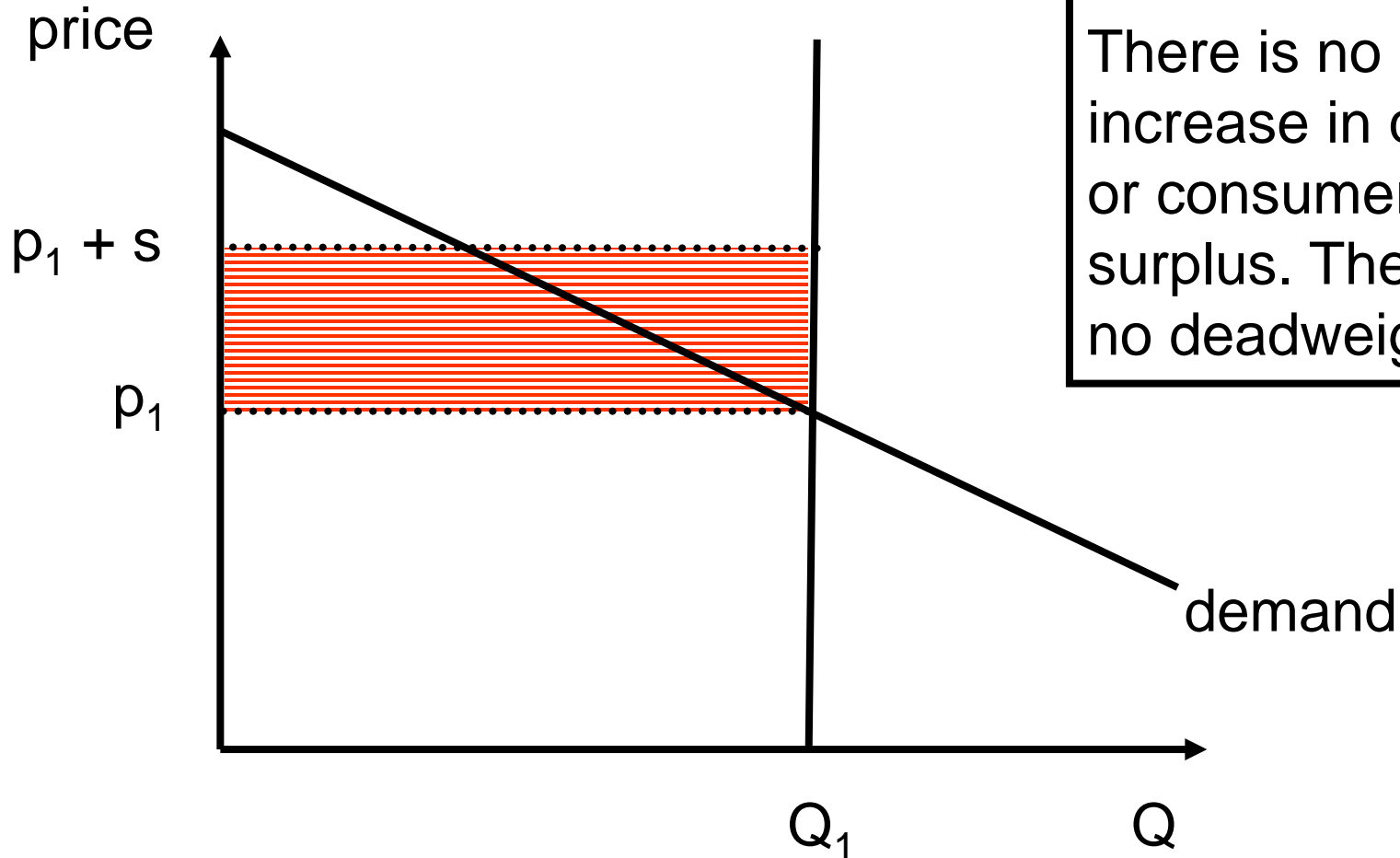


simple welfare economics of a subsidy

gain in producer surplus



With completely inelastic supply all the gain in surplus goes to producers. There is no increase in quantity or consumer surplus. There is no deadweight loss



The gain in consumer and producer surplus from a subsidy is less than the cost of the subsidy.

Who gains from the subsidy on some agricultural products from the Common Agricultural Policy of the European Union?

Simple argument: the cost of subsidy exceeds the benefit to farmers and consumers.

Effects on farmers outside the EU are also important.

The subsidies increases the value of agricultural land.

Land owners gain when the subsidy is introduced, and would lose if the subsidy were removed.

# With supply and demand analysis

Any policy that puts a gap between what consumers pay and producers get moves quantity from the level that maximizes surplus.

Remember the limits of supply and demand analysis assumes price taking.

Consumers know the prices and quality, no asymmetric information

Input prices reflect social costs, so no externalities, perfect competition in the input markets.

Distribution is not taken into account.

# Monopoly

## 6. Monopoly

Up to now I have assumed that firms are price takers.

They are small relative to the market so their output decisions have no effect on prices.

Now look at industries with one firm (monopoly)

Later look at industries with a small number of firms (oligopoly).

# Why are there industries with a small number of firms?

- Economies of scale
  - to be discussed later.
- Legal barriers to entry
  - e.g. licenses, quotas, state monopolies
- Access to a critical input or technology that is not available to other firms,
  - patents
  - secrecy making imitation impossible
  - organizational culture

If  $q > 0$  maximizes profits the first order conditions imply

$$\Pi'(q) = R'(q) - c'(q) = 0 \text{ or}$$

MR marginal revenue =  $R'(q) = c'(q)$  = marginal cost.

This is the **marginal output rule**.

If  $c(0) = 0$  then if  $q = 0$  profits = 0.

Profit maximization then implies  $\Pi(q) = R(q) - c(q) \geq 0$

so if  $q > 0$  maximizes profits

$$\text{average revenue} = R(q)/q = p \geq c(q)/q = AC$$

That is  $p \geq AC$ , the **shutdown rule**.

For a price taking firm marginal revenue  $MR = p$  price

For a firm whose output affects the price it gets using the product rule  $R = pq$  where  $p$  is a function of  $q$

$$MR = \frac{\partial R}{\partial q} = \frac{p \partial q + q \partial p}{\partial q} = p + q \frac{\partial p}{\partial q} = p \left(1 + \frac{q}{p} \frac{\partial p}{\partial q}\right)$$
$$= p \left(1 + \frac{1}{e}\right)$$

where  $e = \frac{p}{q} \frac{\partial q}{\partial p}$  own price elasticity of demand.

$e < 0$  because demand is downward sloping

so marginal revenue  $MR = p \left(1 + \frac{1}{e}\right) < p = \text{price}$



Profit maximization implies  $MC = MR < p$  so  $MC < p$

For a monopoly elasticity, MR and output depend only on demand from buyers.

For a firm in a oligopoly elasticity, MR and output depend on demand from buyers and the output by other firms in the industry.

# Exam mistake with perfect competition models

- Working with perfect competition model
  - Finding industry revenue
  - Finding industry marginal revenue
  - Equating industry marginal revenue to marginal cost.

WRONG

- In perfect competition there is price taking
  - Firm marginal revenue = price
  - Find firm supply as a function of price
  - Equate total supply by all firms to demand

RIGHT

# A simple model of demand

Throughout this section

$Q = \text{industry supply} = \text{demand when } Q = \alpha - \beta p$

where  $p = \text{price}$  so  $p(Q) = a - bQ$

where  $a = \alpha/\beta$  and  $b = 1/\beta$

Assume that  $\alpha > 0$  &  $\beta > 0$  so  $a > 0$  and  $b > 0$ .

For a monopoly total revenue  $p(Q)Q = (a - bQ)Q = aQ - bQ^2$ .

Marginal revenue =

derivative of total revenue with respect to  $Q = a - 2bQ$ .

# Example monopoly with constant returns to scale (CRS) the algebra

A firm is a monopoly when there is no other firm producing the same good.

With CRS the firm's total cost is  $cQ$  where  $c = MC = AC$ .

$$\begin{aligned}\text{If price} = a - bQ \quad \text{profits } p(Q)Q - cQ &= (p(Q) - c)Q \\ &= (a - bQ - c) Q = (a - c)Q - bQ^2.\end{aligned}$$

If  $a \leq c$  profits are negative for all  $Q > 0$ , the firm produces 0.

If  $a > c$  profits  $(a - c)Q - bQ^2$  are a quadratic in  $Q$  with a negative coefficient of  $Q^2$ .

The foc (first order conditions) give a maximum where the derivative is 0, that is  $a - c - 2bQ = 0$ .

The same condition can be got from

MR = marginal revenue = marginal cost =  $c$ .

Total revenue =  $p(Q)Q = (a - bQ)Q = aQ - bQ^2$

differentiating with respect to  $Q$  gives MR =  $a - 2bQ$

so MR = MC gives  $a - 2bQ = c$

or  $a - c - 2bQ = 0$ .

If  $a > c$  profits  $(a - c)Q - bQ^2$  are a quadratic in  $Q$  with a negative coefficient of  $Q^2$ .

The foc (first order conditions) give a maximum where the derivative is 0, that is  $a - c - 2bQ = 0$ .

The same condition can be got from

MR = marginal revenue = marginal cost =  $c$ .

Total revenue =  $p(Q)Q = (a - bQ)Q = aQ - bQ^2$

differentiating with respect to  $Q$  gives  $MR = a - 2bQ$

so  $MR = MC$  gives  $a - 2bQ = c$

or  $a - c - 2bQ = 0$ .

Many people  
don't say this in  
the exam.

From the first order condition  $MR = MC$

$a - c - 2bQ = 0$  so profits are maximized when

$$Q = Q_m = \frac{1}{2} (a - c)/b.$$

price  $p_m = a - b Q_m = a - \frac{1}{2} (a - c) = \frac{1}{2} a + \frac{1}{2} c > c$  because  
 $a > c$ .

profits = total revenue – total cost =  $p_m Q_m - cQ_m$

$$= (p_m - c)Q_m = \frac{1}{2} (a - c)Q_m = \frac{1}{4} (a - c)^2 /b.$$

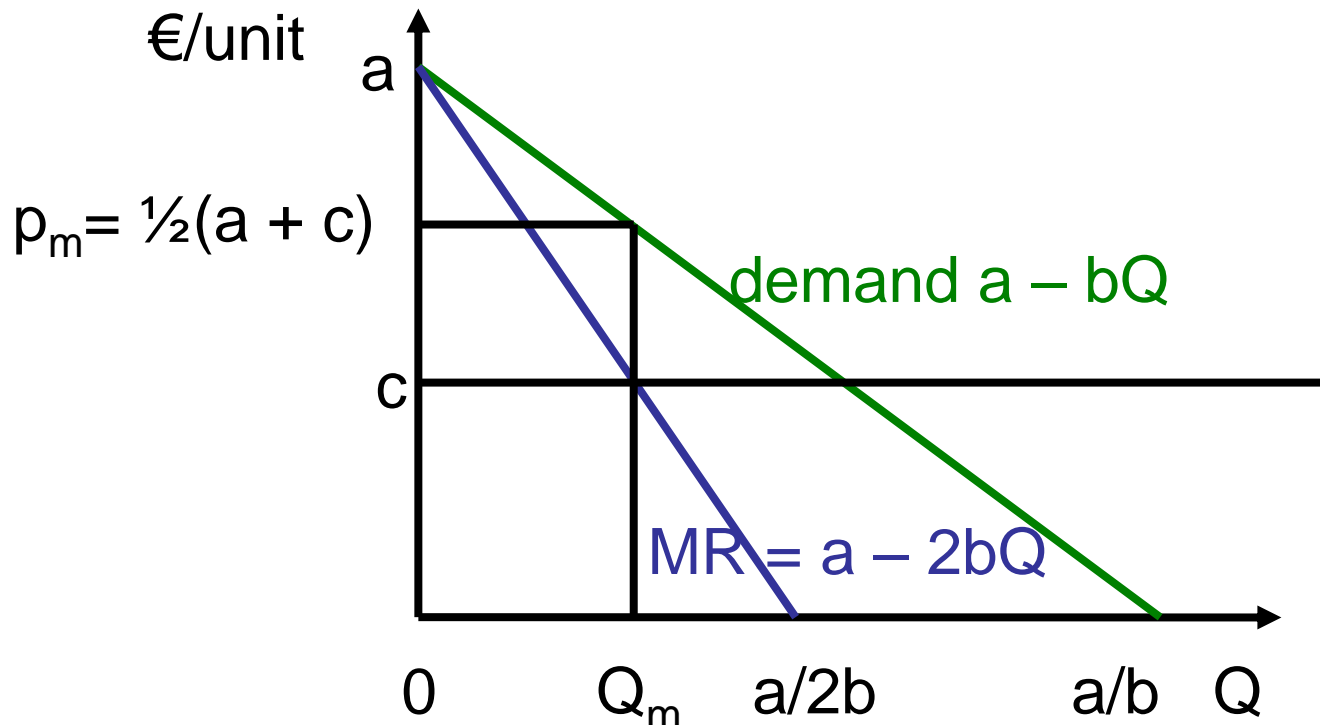
# Monopoly with CRS, the diagram

$$MR = a - 2bQ = c = MC \text{ when } Q = Q_m = \frac{1}{2} (a - c)/b$$

If  $Q < Q_m$   $MR > MC$  profits increase when  $Q$  increases

If  $Q > Q_m$   $MR < MC$  profits decrease when  $Q$  increases

Profit max at  $Q_m$  where  $MR = c = MC$ .





# Monopoly with the cost function $c(q) = F + cq$

Demand gives  $p(Q) = a - bQ$ , profits = total revenue – total cost

$$= p(Q)Q - cQ - F = (a - bQ)Q - cQ - F = (a - c)Q - bQ^2 - F$$

If  $a \leq c$  profits  $< 0$  for all  $Q > 0$  the firm does not produce.

If  $a > c$  profits are a quadratic function of  $Q$  with a negative coefficient on  $Q^2$ .

# Monopoly with the cost function $c(Q) = F + cQ$

If  $Q > 0$  and profits are maximized the foc is  $a - c - 2bQ = 0$

so  $Q = \frac{1}{2} (a - c)/b > 0$

Price  $p(Q) = a - bQ = a - \frac{1}{2} (a - c) = \frac{1}{2} (a + c) = c + \frac{1}{2} (a - c) > c$

Profits =  $p(Q) - c(Q) = (p(Q) - c)Q - F = (a - c)^2 / (4b) - F$ .

If  $F > (a - c)^2 / (4b)$  the firm cannot make a profit so shuts down.

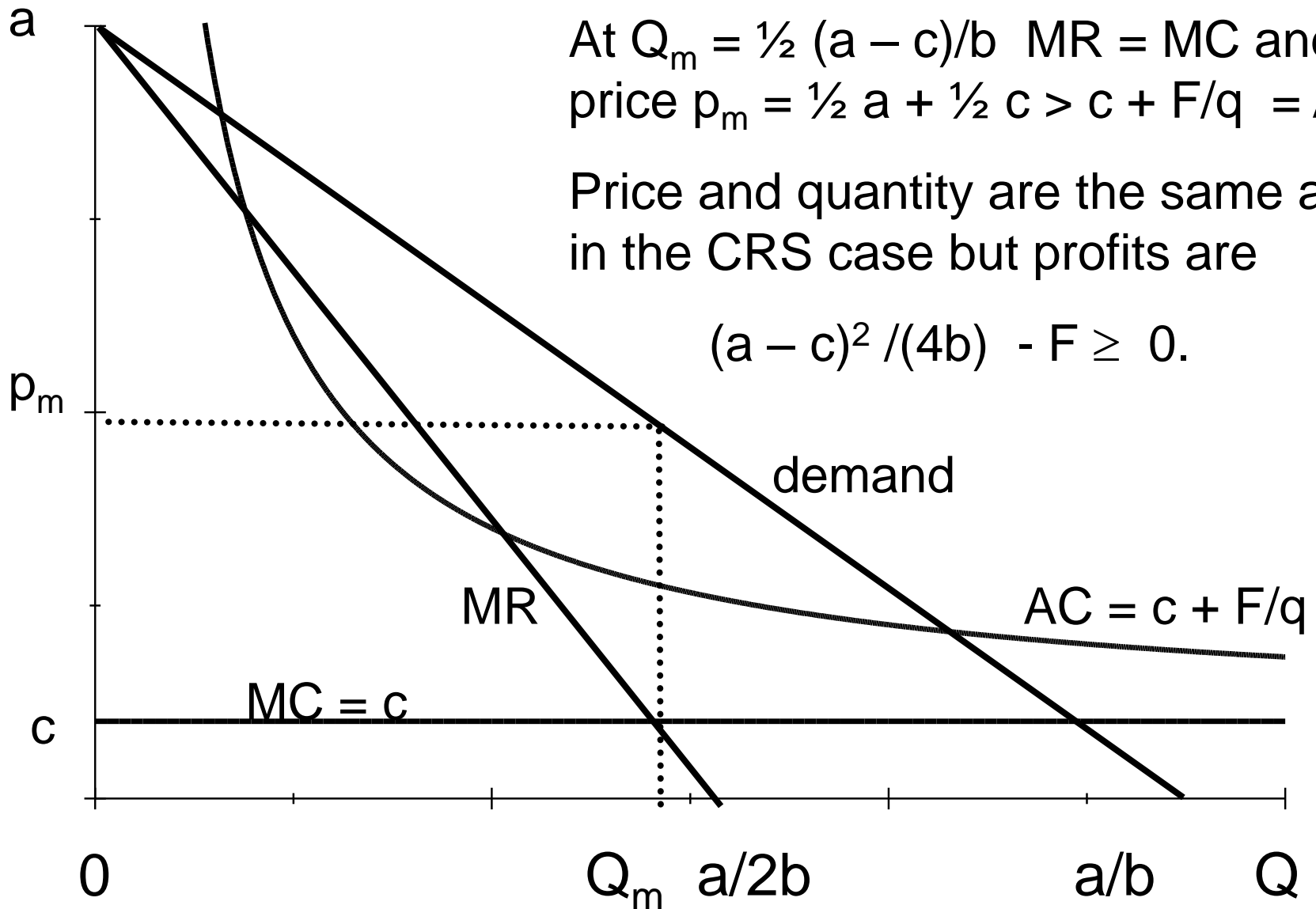
€/unit

Case 1  $(a - c)^2 / (4b) - F \geq 0$

At  $Q_m = \frac{1}{2} (a - c) / b$   $MR = MC$  and price  $p_m = \frac{1}{2} a + \frac{1}{2} c > c + F/q = AC$ .

Price and quantity are the same as in the CRS case but profits are

$(a - c)^2 / (4b) - F \geq 0$ .

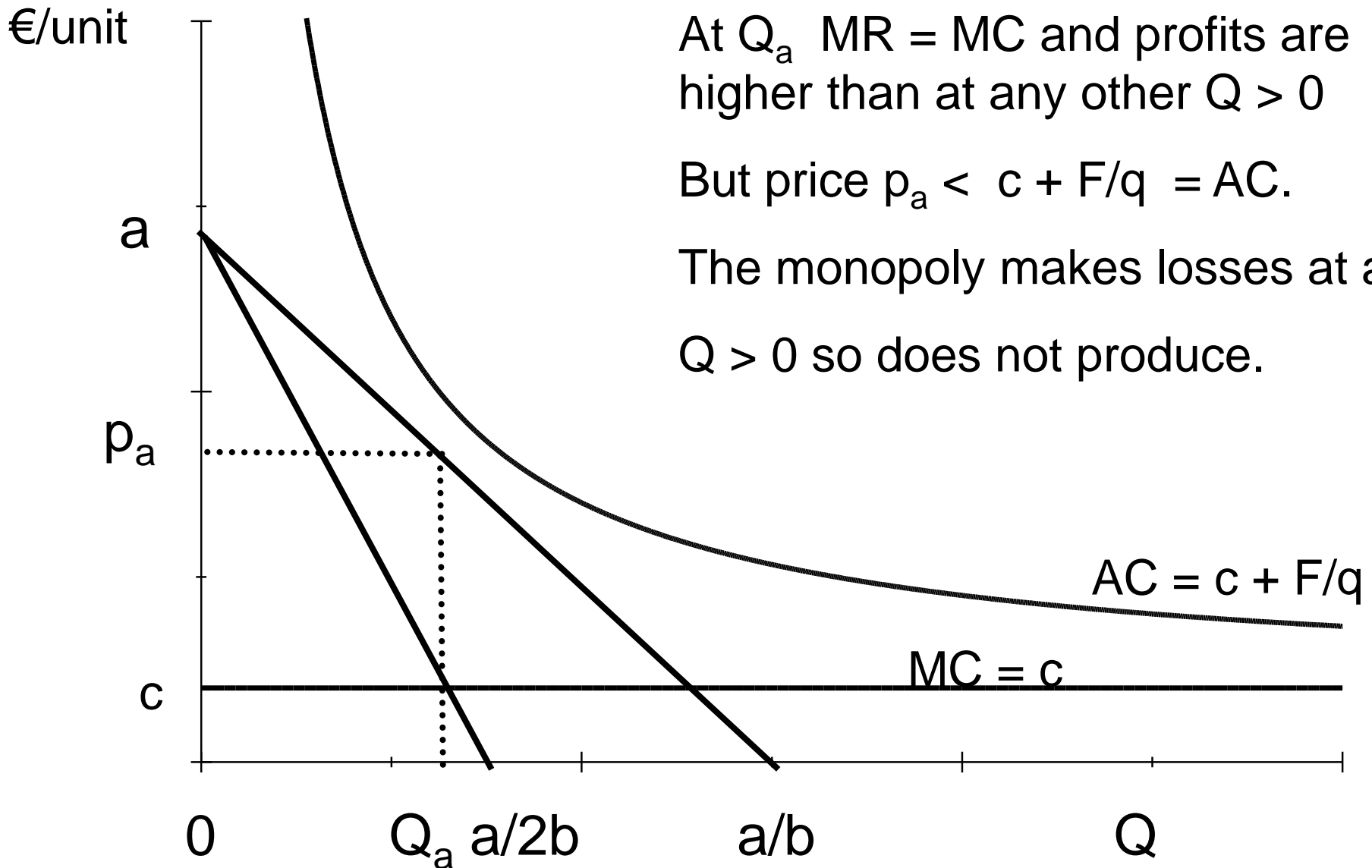


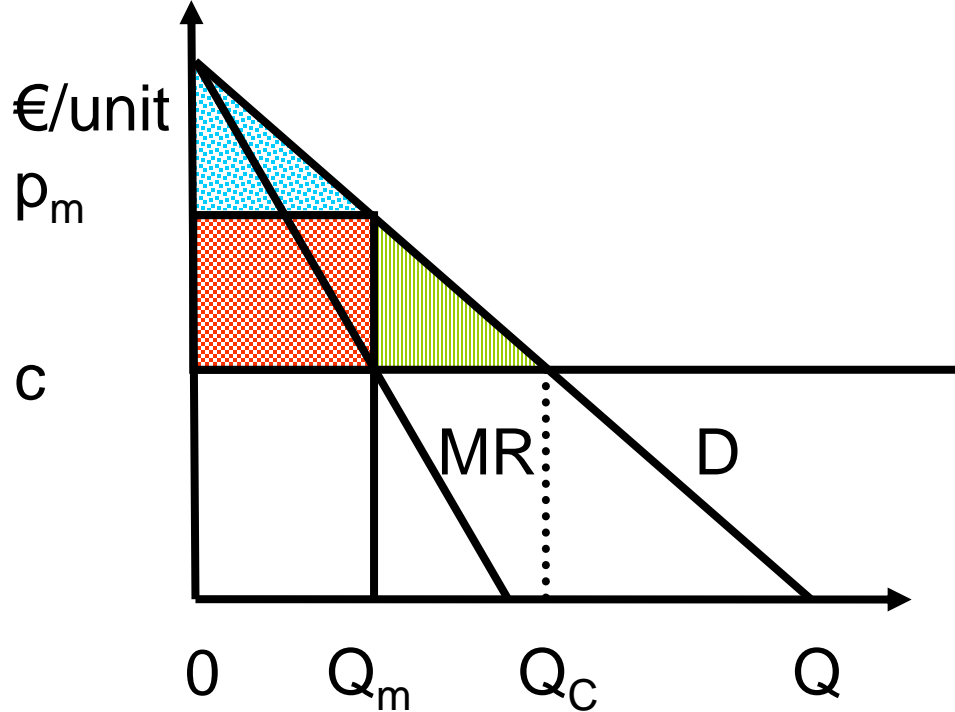
Case 2  $(a - c)^2 / (4b) - F < 0$

At  $Q_a$   $MR = MC$  and profits are higher than at any other  $Q > 0$

But price  $p_a < c + F/q = AC$ .


The monopoly makes losses at all  $Q > 0$  so does not produce.

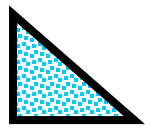




Comparing monopoly and perfect competition there is a deadweight loss due to monopoly. But if there are fixed costs perfect competition is impossible.

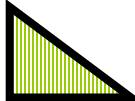
Assume there are no fixed costs, total cost =  $cQ$ .

Monopoly profits =  $(p_m - c) Q_m$  

Consumer surplus 

Compare perfect competition,  $p = c$ ,  $Q = Q_C$

Consumer surplus = entire shaded area.

Loss of consumer surplus due to monopoly – monopoly profits =  
 deadweight loss due to monopoly 

# Cartels

# 7. Cartels

Suppose that there are  $n$  firms in an industry that have a **cartel** agreement to maximize industry profits.

Suppose firm  $i$  has costs  $cq_i + F$ .

Assume  $F > 0$  so there are economies of scale.

Total industry cost

$$= c(q_1 + q_2 + \dots + q_n) + nF = cQ + nF \quad \text{where } Q = q_1 + q_2 + \dots + q_n.$$

Assume as before  $p = a - bQ$ .

$$\text{Industry profits} \quad pQ - cQ - nF = (a - bQ - c)Q - nF$$

Same as monopoly profits except last term is  $nF$  not  $F$ .

# Cartels

$$\begin{aligned}\text{Industry profits} &= (a - bQ - c)Q - nF \\ &= (a - c)Q - bQ^2 - nF.\end{aligned}$$

If  $a \leq c$  the industry cannot make profits.

If  $a > c$  and the industry produces  $Q > 0$

the formula for profits is the same as with monopoly except the fixed cost is  $nF$  not  $F$ .



# Cartels

If  $Q > 0$  maximizes profits  $Q = \frac{1}{2} (a - c) / b$

price  $p = \frac{1}{2} (a + c)$ ,

$p$  and  $Q$  are the same as with a monopoly but

cartel profits  $(a - c)^2 / (4b) - nF$

< monopoly profits  $(a - c)^2 / (4b) - F$ .

If  $n > (a - c)^2 / (4bF)$  the cartel makes losses,  
so at least one firm in the industry makes losses.

As the cartel maximizes industry profits if  $n > (a - c)^2 / (4bF)$   
there is no way the industry can be profitable at any level of  
output.

An industry with  $n > (a - c)^2 / (4bF)$  is impossible when there is  
entry and exit.

A large fixed cost  $F$  is a barrier to entry  
as is low demand, small  $a$  or large  $b$ .

If  $F < (a - c)^2 / (4b) < 2F$  a monopoly can make profits but  
if there are 2 or more firms the industry makes losses.

The industry must be a monopoly.

# What have we achieved?

- Simple model of perfect competition,
  - with limitations but useful policy insights.
- Simple models of monopoly & cartels
  - major limitation: very little discussion of innovation.